Additional Topics

Here are the notes for several additional topics we didn't cover this year. These include

- Large-tip-angle 2D pulses that are linear in rotation angle
- Spectral-spatial spin-echo pulses
- Ultra-short echo time (UTE) pulses, and short T2 contrast generation
- True spin-echo pulses, and spin-echo pulse pairs

Stop by to talk if you are interested in any of these. I'll be around all summer.

Large-tip-angle 2D pulses

- Small rotation solution to the spinor Bloch equation
- Small rotation pulses
- Large-tip-angle 2D pulses that are linear in rotation angle

SMALL ROTATION SOLUTION

BLOCH EQUATION NEGLECTING RELAXATION

$$\frac{d}{dt}\begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix} = \gamma \begin{pmatrix} 0 & \underline{G}\cdot\underline{r} & -B_{1,y} \\ -\underline{G}\cdot\underline{r} & 0 & B_{1,x} \\ B_{1,y} & -B_{1,x} & 0 \end{pmatrix} \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix}$$

WHERE

$$\underline{G} = (G_x, G_y, G_z) \qquad \text{FUNCTION OF } t$$

$$\underline{r} = (x, y, z)$$

NOTE THIS IS LEFT HAND ROTATION.

REWRITE AS

$$\frac{d}{dt}\underline{m} = \omega(\underline{n}\cdot\underline{S})\,\underline{m}$$

WHERE

$$\omega = \gamma\sqrt{(\underline{G}\cdot\underline{r})^2 + B_{1,x}^2 + B_{1,y}^2}$$

$$\underline{n} = \frac{\gamma}{\omega}(B_{1,x},\ B_{1,y},\ B_{1,z})$$

AND $\underline{S} = (S_x, S_y, S_z)$

$$S_x = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \quad S_y = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad S_z = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

THIS IS SO(3) REPRESENTATION.

IN SU(2) THIS CORRESPONDS TO

$$\frac{d}{dt}\underline{\psi} = \frac{i\omega}{2}(\underline{n}\cdot\underline{\sigma})\,\underline{\psi}$$

WHERE ω AND $\underline{n}$ ARE THE SAME, AND

$\underline{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

SUBSTITUTING

$$\frac{d}{dt}\underline{\psi} = \frac{i\omega}{2}\left(\frac{\gamma}{\omega}B_{1,x}\sigma_x + \frac{\gamma}{\omega}B_{1,y}\sigma_y + \frac{\gamma}{\omega}\underline{G}\cdot\underline{r}\,\sigma_z\right)\underline{\psi}$$

$$= \frac{i\gamma}{2}\left(B_{1,x}\sigma_x + B_{1,y}\sigma_y + \underline{G}\cdot\underline{r}\,\sigma_z\right)\underline{\psi}$$

$$\frac{d}{dt}\underline{\psi} = \frac{i\gamma}{2}\begin{pmatrix} \underline{G}\cdot\underline{r} & B_1^*(t) \\ B_1(t) & -\underline{G}\cdot\underline{r} \end{pmatrix}\underline{\psi}$$

RECALL THAT

$$\underline{\psi} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

SO

$$\frac{d}{dt}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \frac{i\gamma}{2}\begin{pmatrix} \underline{G}(t)\cdot\underline{r} & B_1^*(t) \\ B_1(t) & -\underline{G}(t)\cdot\underline{r} \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

WRITING THESE AS TWO EQUATIONS

$$\dot{\alpha} = \frac{i\gamma}{2}\underline{G}(t)\cdot\underline{r}\ \alpha + \frac{i\gamma}{2}B_1^*(t)\ \beta$$

$$\dot{\beta} = \frac{i\gamma}{2}B_1(t)\ \alpha - \frac{i\gamma}{2}\underline{G}(t)\cdot\underline{r}\ \beta$$

HERE WE MAKE <u>SMALL ANGLE APPROXIMATION</u>.

THE ROTATION ANGLE ϕ IS SMALL, SO

$$\beta = -i(n_x + i n_y)\sin\phi/2$$

$$\simeq 0$$

THEN WE CAN SOLVE THE α EQUATION

$$\dot{\alpha} = \frac{i\gamma}{2}\, \underline{G}(t)\cdot\underline{\Gamma}\, \alpha$$

$$\left(\dot{\alpha} - \frac{i\gamma}{2}\, \underline{G}(t)\cdot\underline{\Gamma}\, \alpha\right) = 0$$

$$\frac{d}{dt}\left(\alpha\, e^{-\frac{i}{2}\underline{\Gamma}\cdot\int_{-\infty}^{t}\gamma\, \underline{G}(s)\,ds}\right) = 0$$

$$\alpha\, e^{-\frac{i}{2}\underline{\Gamma}\cdot\int_{-\infty}^{t}\gamma\, \underline{G}(s)\,ds} = C$$

$$\alpha(t) = C\, e^{\frac{i}{2}\underline{\Gamma}\cdot\gamma\int_{-\infty}^{t}\underline{G}(s)\,ds}$$

$$= C\, e^{-i2\pi\,\underline{\Gamma}\cdot\frac{\underline{k}(-\infty,t)}{2}}$$

WHERE

$$\underline{k}(\tau,t) = -\frac{\gamma}{2\pi}\int_{\tau}^{t}\underline{G}(s)\,ds$$

THE CONSTANT $C = 1$, SINCE NO ROTATION IS $\phi = 0$

$$\alpha = \cos\phi/2 - i n_z \sin\phi/2$$
$$= 1$$

SO

$$\alpha(t) = e^{-i2\pi\frac{1}{2}\left(\underline{\Gamma}\cdot\underline{k}(-\infty,t)\right)}$$

OR

$$\alpha(t) = e^{\frac{i}{2}\gamma\left(\underline{\Gamma}\cdot\int_{-\infty}^{t}\underline{G}(s)\,ds\right)}$$

WE CAN USE α TO SOLVE FOR β

$$\dot{\beta} = \frac{i\gamma}{2} B_1(t)\,\alpha - \frac{i\gamma}{2}\,\underline{G}(t)\cdot\underline{r}\,\beta$$

$$\dot{\beta} + \frac{i\gamma}{2}\,\underline{G}(t)\cdot\underline{r}\,\beta = \frac{i\gamma}{2} B_1(t)\,\alpha$$

$$\dot{\beta} + \frac{i\gamma}{2}\,\underline{G}(t)\cdot\underline{r}\,\beta = \frac{i\gamma}{2} B_1(t)\,\underbrace{e^{\frac{i}{2}\gamma\left(\underline{r}\cdot\int_{-\infty}^{t}\underline{G}(s)\,ds\right)}}_{\text{SAME AS INTEGRATING FACTOR}}$$

MULTIPLY BY INTEGRATING FACTOR

$$e^{\frac{i\gamma}{2}\int_{-\infty}^{t}\underline{G}(s)\cdot\underline{r}\,ds}$$

WE GET

$$\frac{d}{dt}\left(\beta\, e^{\frac{i\gamma}{2}\int_{-\infty}^{t}\underline{G}(s)\cdot\underline{r}\,ds}\right)$$

$$= \frac{i\gamma}{2} B_1(t)\, e^{i\gamma\int_{-\infty}^{t}\underline{G}(s)\cdot\underline{r}\,ds}$$

AFTER INTEGRATING

$$\beta\, e^{\frac{i\gamma}{2}\int_{-\infty}^{t}\underline{G}(s)\cdot\underline{r}\,ds}$$

$$= \frac{i\gamma}{2}\int_{-\infty}^{t} B_1(\tau)\, e^{i\gamma\int_{-\infty}^{\tau}\underline{G}(s)\cdot\underline{r}\,ds}\, d\tau$$

SO

$$\beta(t) = \frac{i\gamma}{2} e^{-\frac{i\gamma}{2}\int_{-\infty}^{t} \underline{G}(s)\cdot \underline{r}\, ds} \int_{-\infty}^{t} B_1(\tau)\, e^{i\gamma \int_{-\infty}^{\tau} \underline{G}(s)\cdot \underline{r}\, ds}\, d\tau$$

$$= \frac{i\gamma}{2} e^{\frac{i\gamma}{2}\int_{-\infty}^{t} \underline{G}(s)\cdot \underline{r}\, ds} \int_{-\infty}^{t} B_1(\tau)\, e^{-i\gamma \int_{\tau}^{t} \underline{G}(s)\cdot \underline{r}\, ds}\, d\tau$$

AGAIN

$$\underline{k}(\tau, t) = \frac{-\gamma}{2\pi} \int_{\tau}^{t} \underline{G}(s)\, ds$$

SO

$$\underline{\beta(t) = \frac{i\gamma}{2} e^{-i2\pi\left(\frac{\underline{r}\cdot \underline{k}(-\infty,t)}{2}\right)} \int_{-\infty}^{t} B_1(\tau)\, e^{i2\pi\, \underline{k}(\tau,t)\cdot \underline{r}}\, d\tau}$$

AND

$$\underline{\alpha(t) = e^{-i2\pi\left(\frac{\underline{r}\cdot \underline{k}(-\infty,t)}{2}\right)}}$$

THEN m_{xy} IS

$$m_{xy} = 2\alpha^{*}\beta$$

$$= 2\left(e^{-i2\pi\left(\frac{\underline{r}\cdot \underline{k}(-\infty,t)}{2}\right)}\right)^{*}\left(\frac{i\gamma}{2} e^{-i2\pi\left(\frac{\underline{r}\cdot \underline{k}(-\infty,t)}{2}\right)}\right) \int_{-\infty}^{t} B_1(\tau)\, e^{i2\pi\, \underline{k}(\tau,t)\cdot \underline{r}}\, d\tau$$

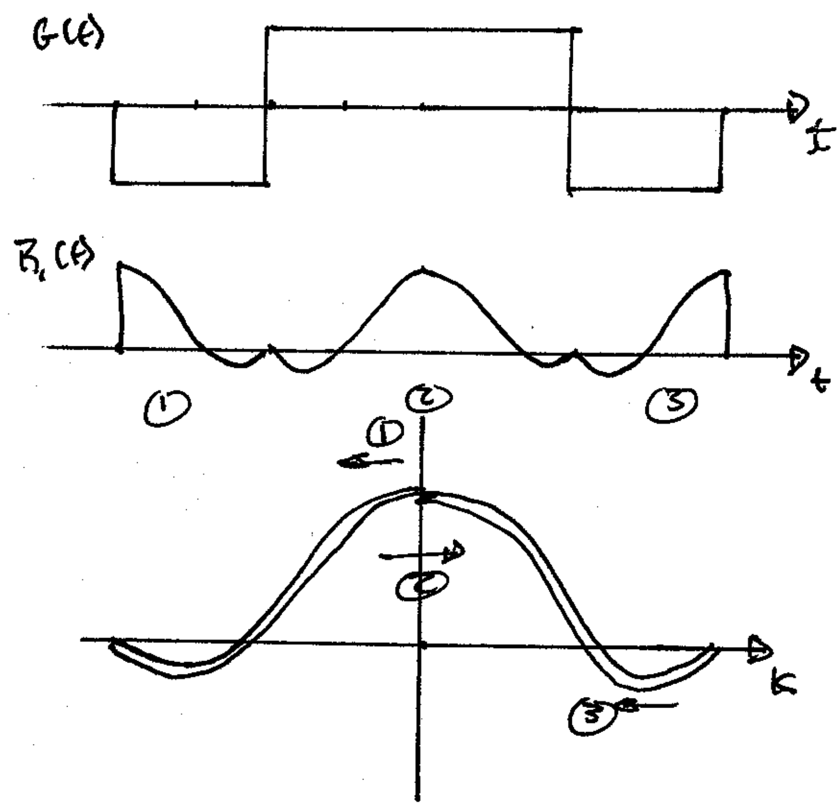
SO

$$\underline{m_{xy}(t) = i\gamma \int_{-\infty}^{t} B_1(\tau)\, e^{i 2\pi \underline{k}(\tau,t)\cdot \underline{r}}\, d\tau}$$

WHICH IS THE SAME AS BEFORE.

INHERENTLY REFOCUSED PULSES

IMPORTANT SPECIAL CASE WHEN GRADIENTS INTEGRATE TO ZERO, B_1 CONJUGATE SYMMETRIC



IN THIS CASE

$$\underline{k}(-\infty, t) = 0$$

SO

$$\alpha(t) = e^{-i 2\pi \frac{\underline{r} \cdot \underline{k}(-\infty, t)}{2}}$$

$$= 1$$

AND

$$\beta(t) = \frac{i\gamma}{2} \underbrace{e^{-i2\pi \frac{\underline{r}\cdot\underline{k}(-\infty,t)}{2}}}_{1} \int_{-\infty}^{t} B_1(\tau) e^{i2\pi \underline{k}(\tau,t)\cdot\underline{r}} d\tau$$

$$= \frac{i\gamma}{2} \underbrace{\int_{-\infty}^{t} B_1(\tau) e^{i2\pi \underline{k}(\tau,t)\cdot\underline{r}} d\tau}_{\text{REAL}}$$

IF $B_1(\tau)$ PRODUCES HERMITIAN SYMMETRIC WEIGHTING, AND $\underline{k}(\tau,t)$ IS SYMMETRIC, THE INTEGRAL IS REAL, AND $\beta(t)$ IS IMAGINARY.

RECALL

$$\beta = -i(n_x + i n_y)\sin\phi/2$$

SO THAT THIS CORRESPONDS TO A ROTATION ABOUT x, AND

$$\phi_x(\underline{r},t) = 2\sin^{-1}(-\beta(\underline{r},t))$$

FOR SMALL ANGLES $\sin^{-1}x = x$, AND

$$\phi(\underline{r},t) = -2\frac{\gamma}{2}\int_{-\infty}^{t} B_1(\tau) e^{i2\pi \underline{k}(\tau,t)\cdot\underline{r}} d\tau$$

$$\underline{\phi(\underline{r},t) = -\int_{-\infty}^{t} \gamma B_1(\tau) e^{i2\pi \underline{k}(\tau,t)\cdot\underline{r}} d\tau}$$

THE <u>ROTATION ANGLE</u> IS FOURIER TRANSFORM OF WEIGHTED K-SPACE TRAJECTORY.

THIS HOLDS FOR <u>ANY</u> INITIAL MAGNETIZATION, (SINCE WE SOLVED FOR ROTATION)

<u>SIGNIFICANCE:</u>

LARGE TIP ANGLE PULSES CAN BE BUILT UP FROM SMALL ROTATIONS ABOUT A COMMON AXIS.

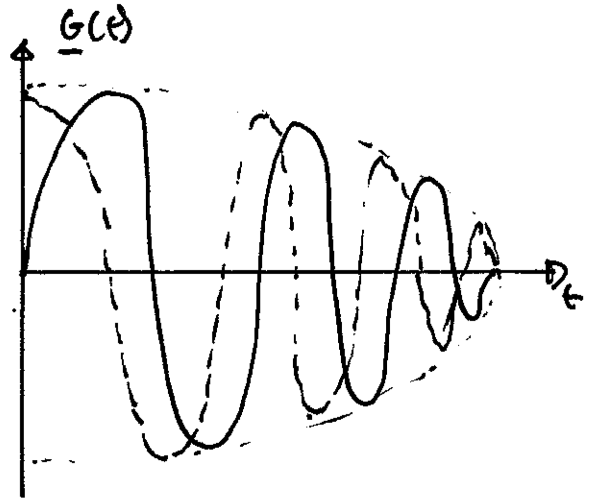
<u>TWO IMPORTANT CASES</u>

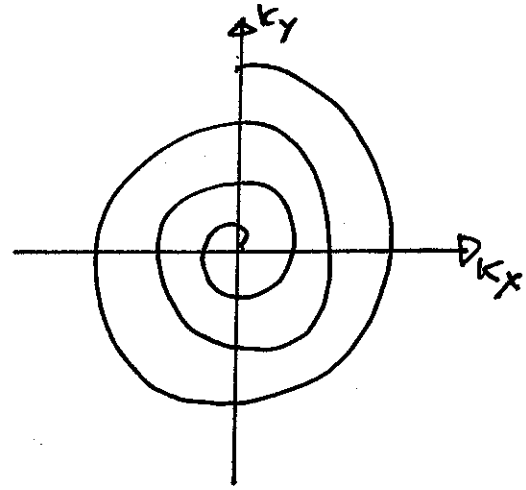
1) 1D SPATIALLY SELECTIVE ROTATIONS FOR SPECTRAL-SPATIAL INVERSIONS, SPIN-ECHOES (NEXT TIME)

2) 2D AND 3D ROTATIONS THAT ARE BUILT UP IN SPATIAL FREQUENCY SPACE

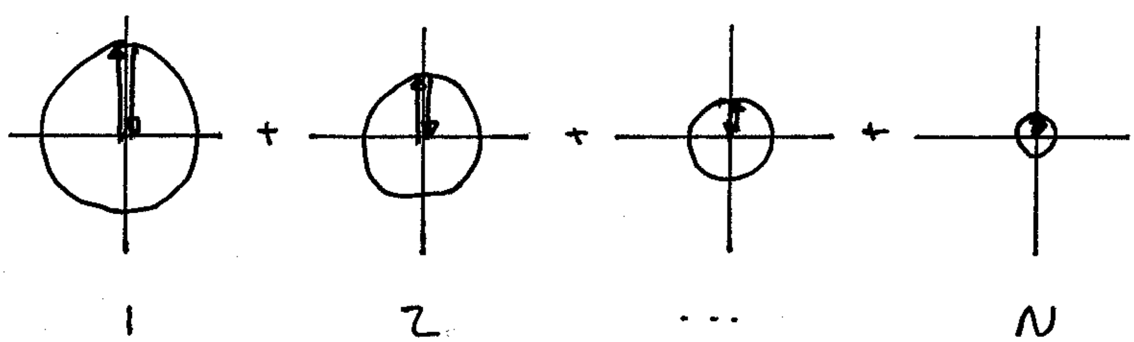
EXAMPLE OF CASE 2

SPIRAL INVERSION OR SPIN-ECHO PULSE





APPROXIMATE N-TURN SPIRAL BY N CIRCLES PLUS BLIPS



THE j^{th} TURN OF THE SPIRAL PRODUCES

$$\phi_j(\underline{r}, t_j) = -\int_{t_{j-1}}^{t_j} \gamma B_1(\tau)\, e^{i 2\pi \underline{k}(\tau, t_j)\cdot \underline{r}}\, d\tau$$

ALL ROTATIONS ARE ABOUT THE SAME AXIS, SO THEY ADD. THE TOTAL ROTATION IS

$$\phi(\underline{r}, t) = \sum_{j=1}^{N} \phi_j(\underline{r}, t_j)$$

$$= \sum_{j=1}^{N} \left(-\int_{t_{j-1}}^{t_j} \gamma B_1(\tau)\, e^{i 2\pi \underline{k}(\tau, t_j)\cdot \underline{r}}\, d\tau \right)$$

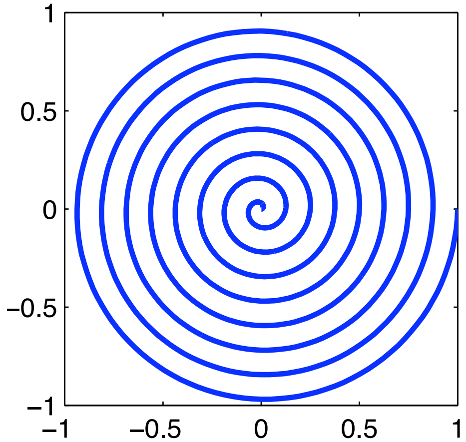
$$= -\int_0^t \gamma B_1(\tau)\, e^{i 2\pi \underline{k}(\tau, t)\cdot \underline{r}}\, d\tau$$

THE ROTATION PRODUCED BY A LARGE-TIP-ANGLE PULSE IS THE FOURIER TRANSFORM OF WEIGHTED K-SPACE TRAJECTORY <u>PROVIDED</u>:

1) THE K-SPACE TRAJECTORY CAN BE DECOMPOSED INTO SYMMETRIC SEGMENTS ABOUT THE ORIGIN

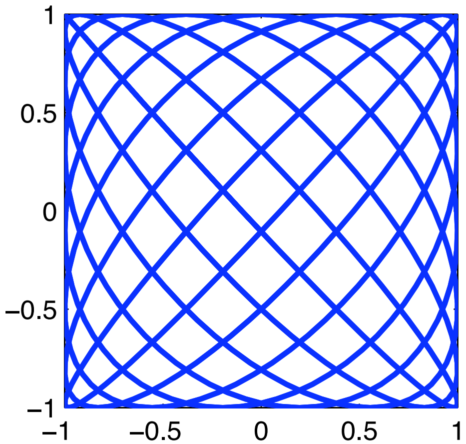
2) THE RF PULSE PRODUCES CONJUGATE SYMMETRIC K-SPACE WEIGHTING

Linear Large-Flip-Angle Trajectories



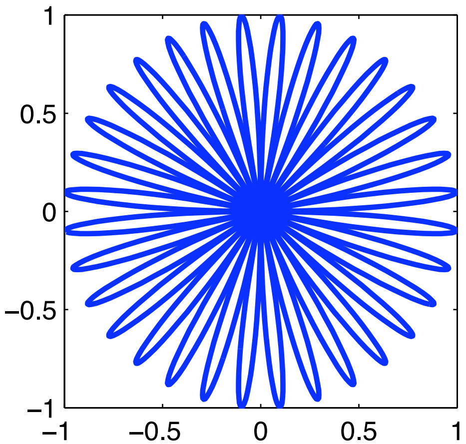
Spiral

$$k_x = t\cos(2\pi 8t)$$
$$k_y = t\sin(2\pi 8t)$$



Lissajou

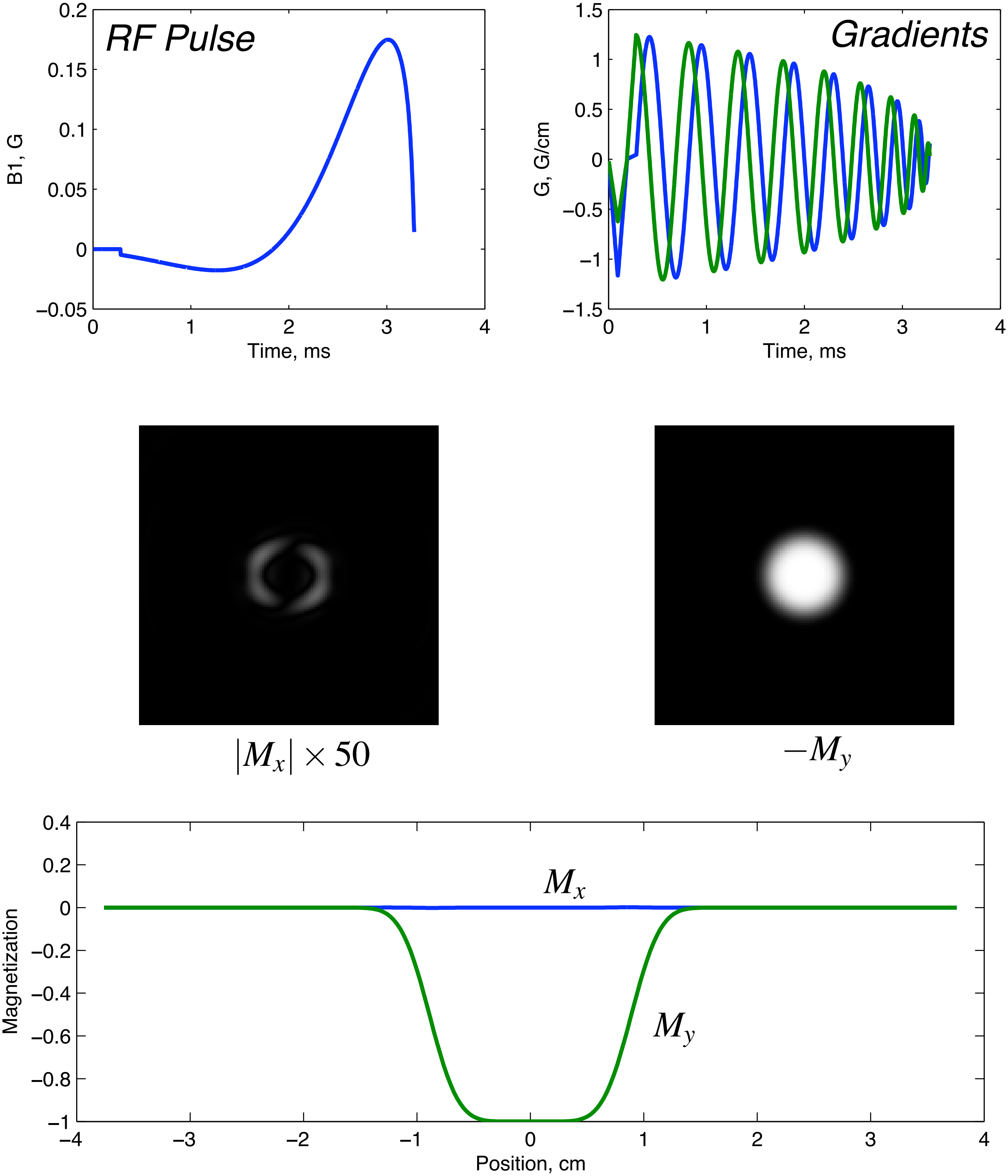
$$k_x = \cos(2\pi 15t)$$
$$k_y = \cos(2\pi 16t)$$

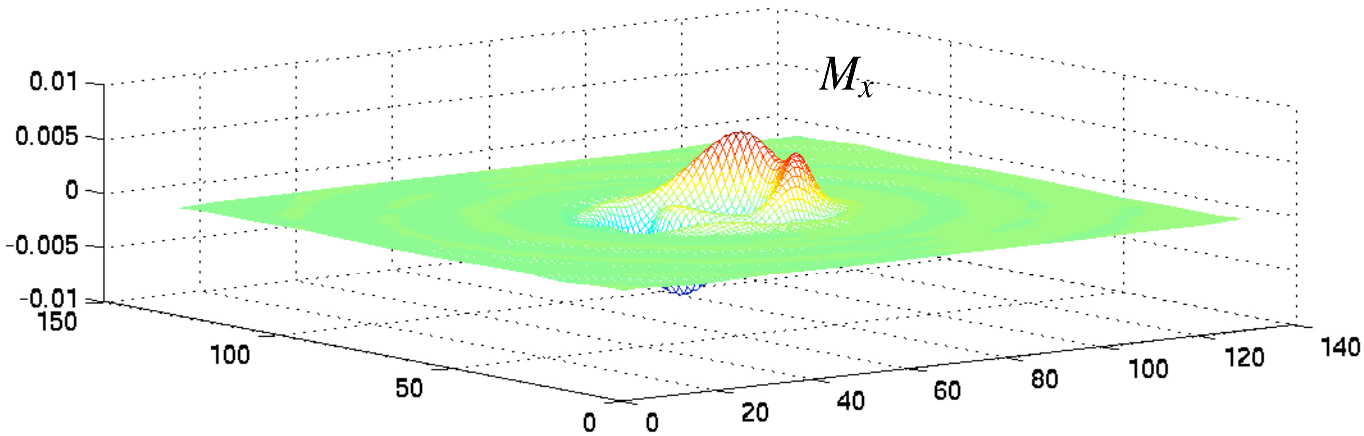


Petal

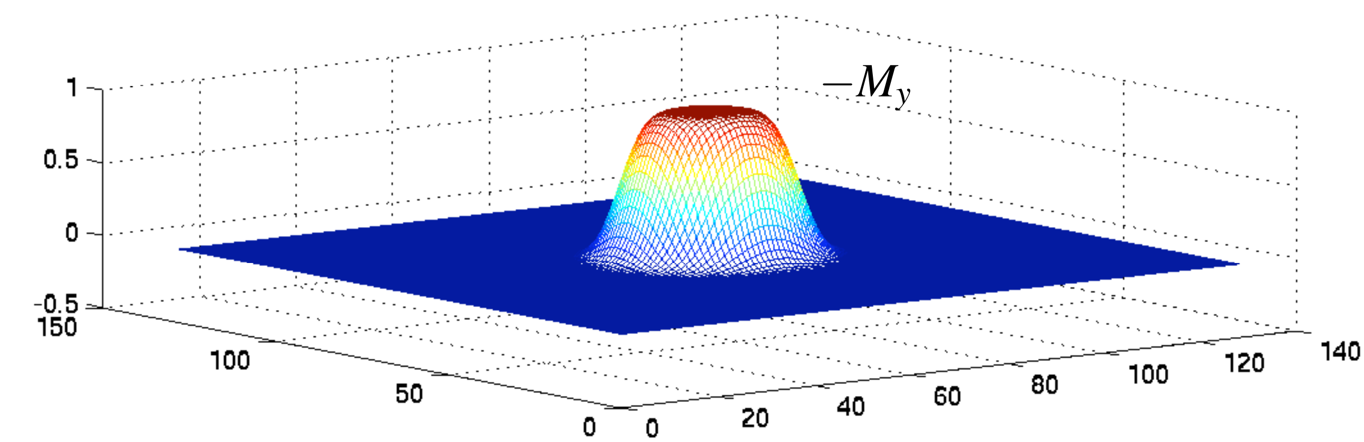
$$k_x = \cos(2\pi t)\sin(2\pi 16t)$$
$$k_y = \sin(2\pi t)\sin(2\pi 16t)$$

Spiral Spin-Echo Pulse





M_x
0.01
0.005
0
-0.005
-0.01
150
100
50
0
0
20
40
60
80
100
120
140



$-M_y$
1
0.5
0
-0.5
150
100
50
0
0
20
40
60
80
100
120
140

Large-tip-angle spectral-spatial pulses

- Why scaling small tip angle spectral-spatial pulses to large tip angles doesn't work
- Separable pulses with linear subpulses and an SLR window
- Full 2D non-linear designs

LARGE-FLIP-ANGLE SPECTRAL-SPATIAL PULSES

OFTEN CALLED EPSE PULSES, FOR

ECHO-PLANAR SPIN-ECHO PULSES

SINCE SPECTRAL SPATIAL PULSES MUST BE BASED ON EPI TRAJECTORY.

OUTLINE

1) WHY SCALING SPECTRAL-SPATIAL EXCITATION PULSES DOESN'T WORK WELL.

2) SEPARABLE 1D NON-LINEAR, 1D LINEAR DESIGNS

3) FULL 2D NON-LINEAR DESIGNS

FIRST

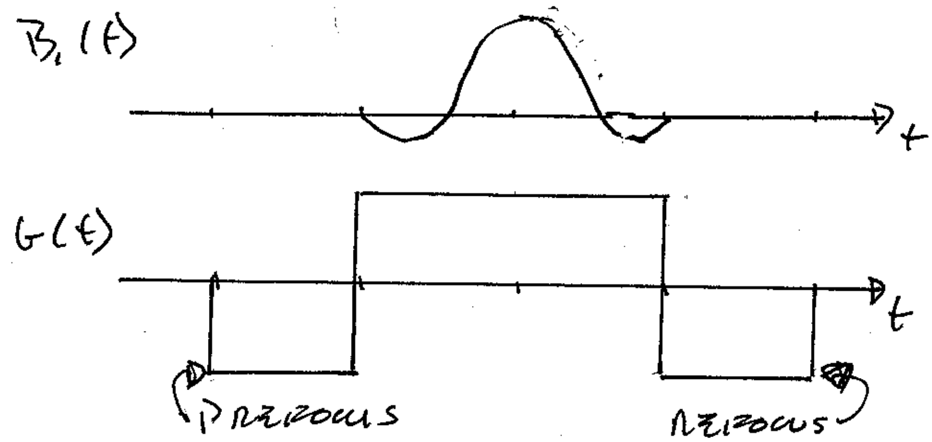
GRADIENT WAVEFORMS

-AND-

INHERENTLY REFOCUSED PULSES

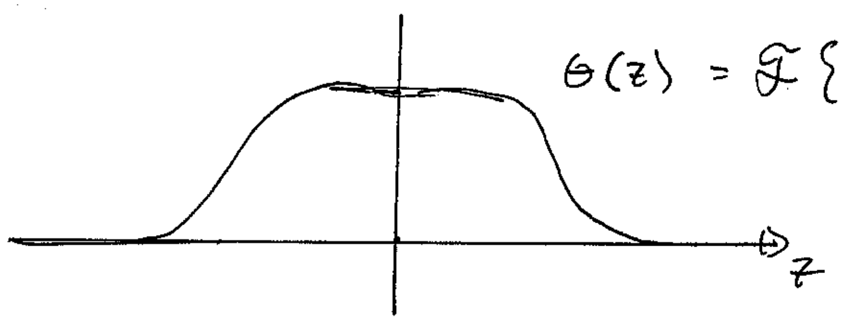
RECALL FROM LAST TIME

AN RF PULSE WITH THE WAVEFORMS:

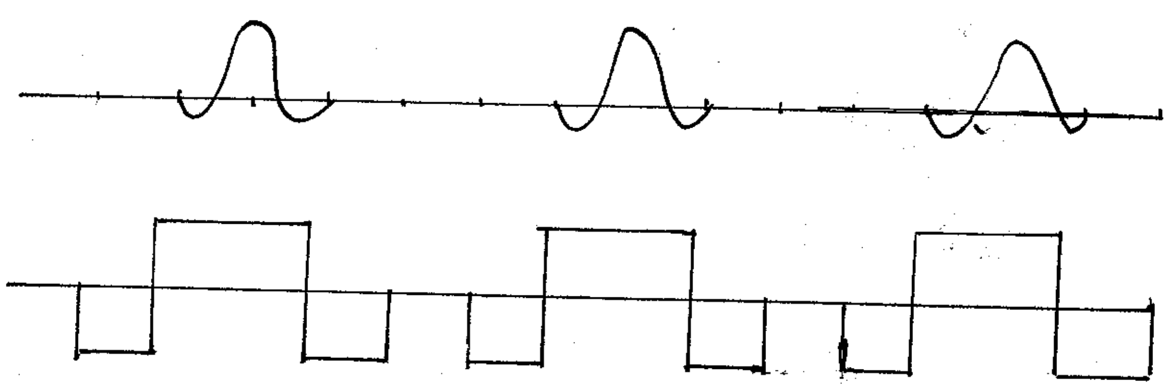


PRODUCES A ROTATION ABOUT THE RF AXIS.

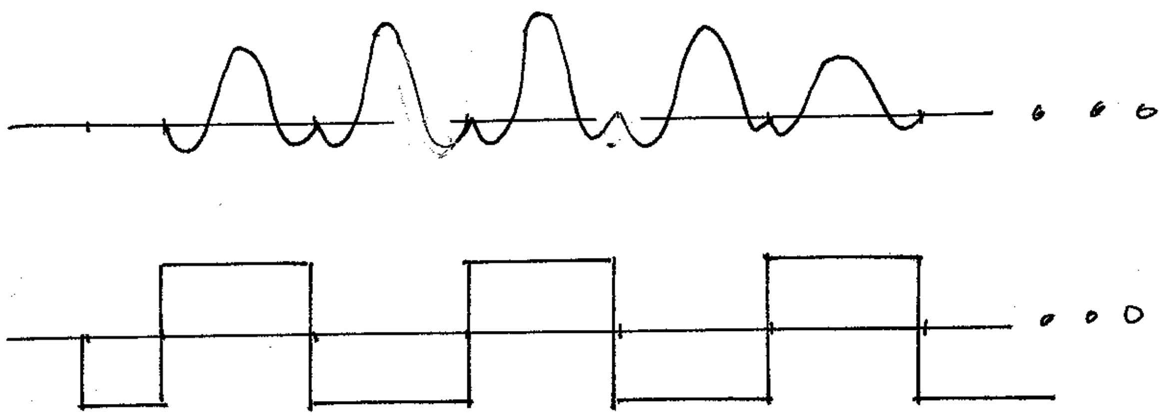
$$\theta(z) = \mathcal{F}\{B_1(t)\}\Big|_{f=\frac{\gamma}{2\pi}Gz}$$



WE CAN BUILD UP LARGER-FLIP-ANGLE PULSES
AS A COMBINATION OF THESE ROTATIONS

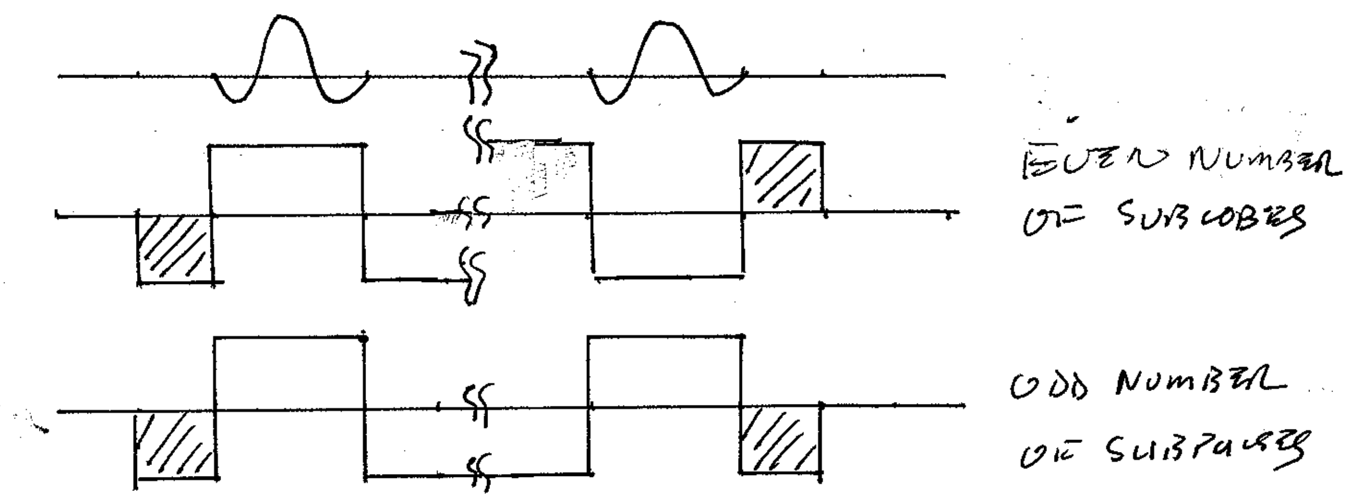


INVERTING ALTERNATE PULSES, AND COMPRESSING:



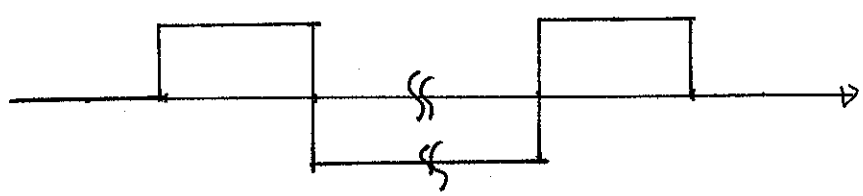
PRE AND REPHASE LOBES CANCEL.

IF WE CONSIDER THE FIRST AND LAST LOBES

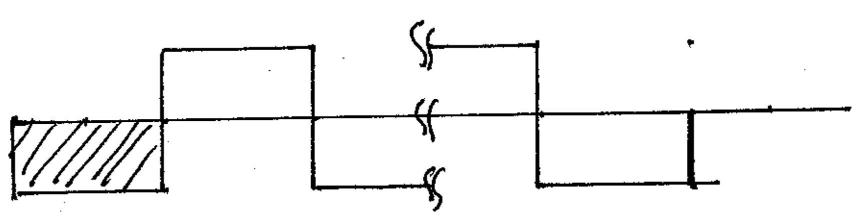


IF THE RF PULSE IS A 180, A GRADIENT LOBE +G BEFORE THE 180 IS THE SAME AS A GRADIENT LOBE −G AFTER THE 180.

HENCE WITH AN ODD NUMBER OF SUBLOBES, THE PRE/RE FOCUSING GRADIENTS CANCEL

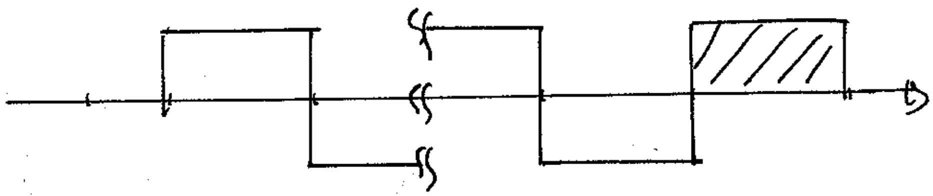


WITH AN EVEN NUMBER OF SIDELOBES, THEY DON'T CANCEL. WE CAN HAVE PRE/RE FOCUSING GRADIENTS (ABOVE) OR ONLY PREFOCUSING GRADIENTS



PREFOCUS ONLY

ONLY REFOCUSING GRADIENTS



REFOCUS ONLY

WHATEVER YOU CHOOSE, EVEN NUMBER OF SUBLOBES IS NOT SYMMETRIC. THIS IS SURPRISING!

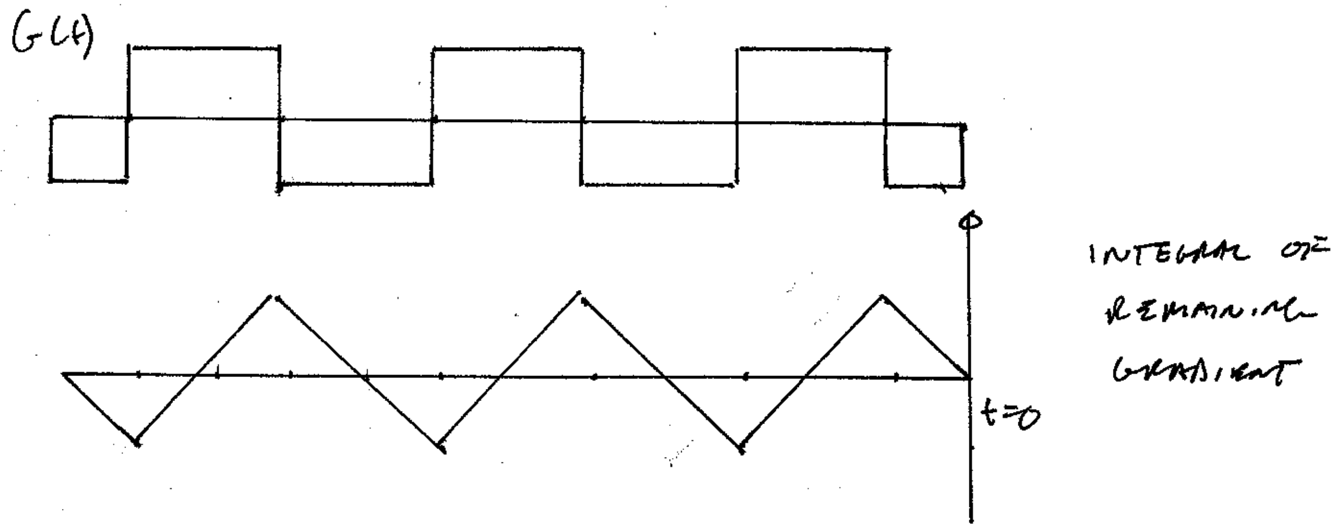
RF PULSE DESIGN

LINEAR DESIGN

CHOOSE K-SPACE WEIGHTING THAT IS FOURIER TRANSFORM OF DESIRED PROFILE. (WINDOWED SINCS)

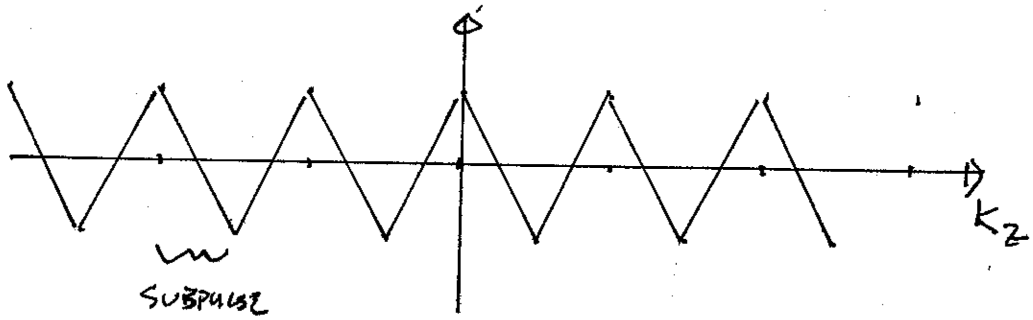
LINEAR DESIGN WORKS FOR SPIRAL PULSES (LAST TIME). DOES IT WORK HERE?

LOOK AT K-SPACE TRAJECTORY



<u>NOT</u> HERMITIAN SYMMETRIC ABOUT ORIGIN

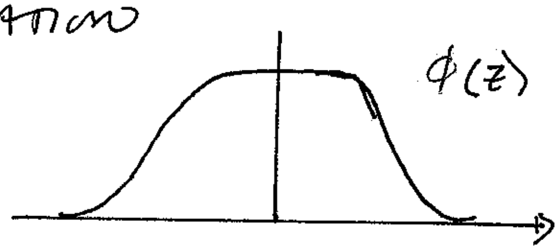
EVEN IF WE SHIFT ORIGIN (SPATIAL PULSE, REFOCUSING GRADIENT) STILL NOT HERMITIAN SUBPULSES.



EACH SUBPULSE NOT SYMMETRIC, ROTATIONS DON'T ADD.

WHAT HAPPENS WHEN YOU PLAY A LINEAR DESIGNED PULSE AT 180°?

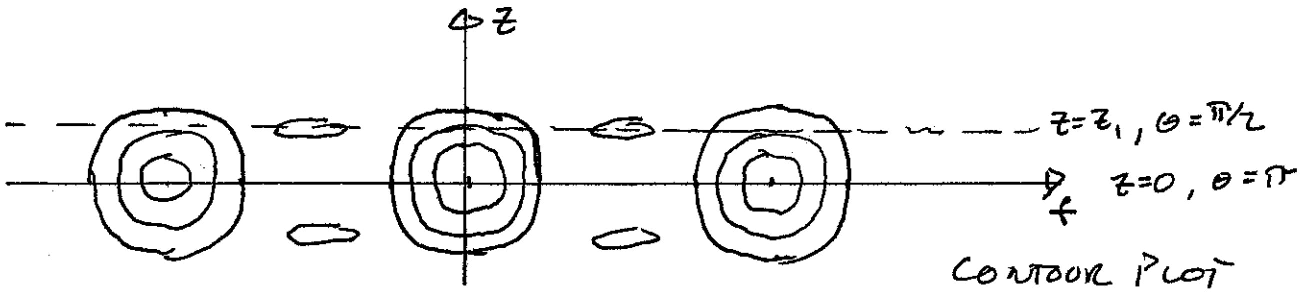
EACH SUBPULSE PRODUCES A SPATIALLY VARYING ROTATION



IN THE MIDDLE IT PRODUCES ITS DESIRED ROTATION, IN THE TRANSITION BAND, SOMETHING LESS

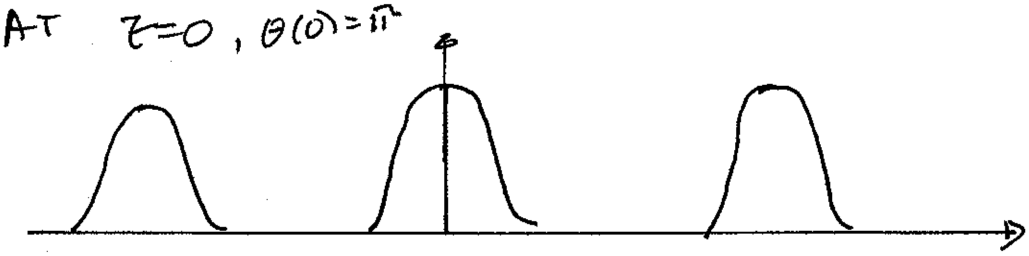
AT $z=0$, THE RESULT IS A WINDOWED SINC SPECTRAL PULSE PLAYED AT 180

AT z IN THE TRANSITION BAND, THE SAME SPECTRAL PULSE IS PLAYED AT A LOWER AMPLITUDE



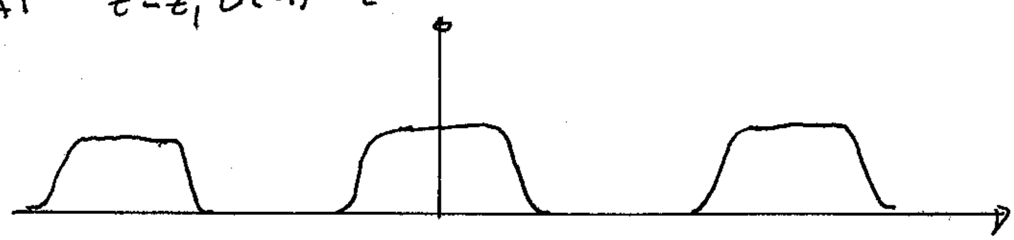
CONTOUR PLOT

AT $z=0,\ \theta(0)=\pi$



NARROWED PROFILE
WINDOWED SINC
NOT GOOD 180

AT $z=z_1\ \theta(z_1)=\pi/2$



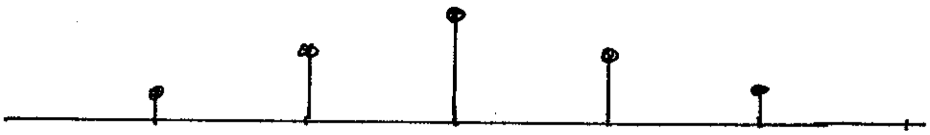
GOOD PROFILE
WINDOWED SINC
GOOD 90

OK, WHY NOT FIX SPECTRAL WINDOW?

NON-LINEAR 2D SEPARABLE DESIGN

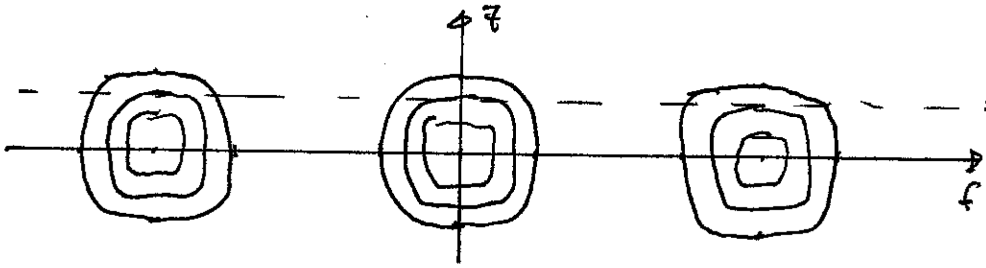
AT $z=0$, WE ARE PLAYING AN N-SAMPLE HARD PULSE EXCITATION

π HARD PULSE



DESIGN WINDOW WITH SLR, STILL USE WINDOWED SINC SUBPULSES

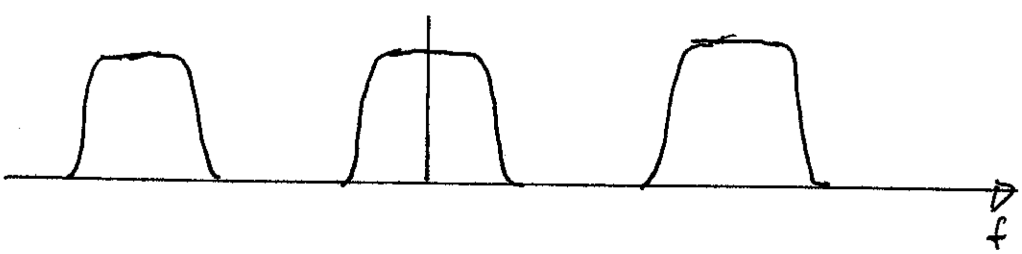
SPIN ECHO
$m_y(z,f) = \beta^2(z,f)$



$z = z_1$, $\theta = \pi/2$

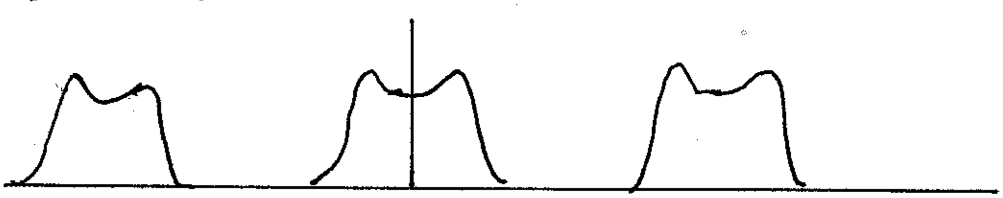
$z=0$, $\theta = \pi$

AT $z=0$, $\theta(0) = \pi$



NICE SQUARE RESPONSE
SLR HARD PULSE IS GOOD π

AT $z=z_1$ $\theta(z_1) = \pi/2$

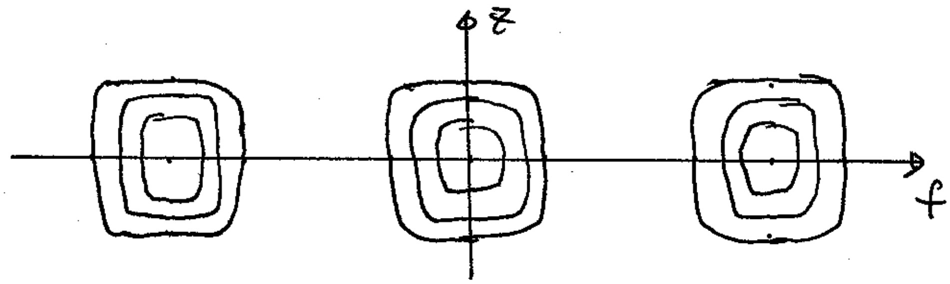


DISTORTED RESPONSE
π PLAYED AT HALF AMPLITUDE
NOT A GOOD $\pi/2$

WE NEED A DIFFERENT SPECTRAL PULSE AT EACH SPATIAL POSITION!

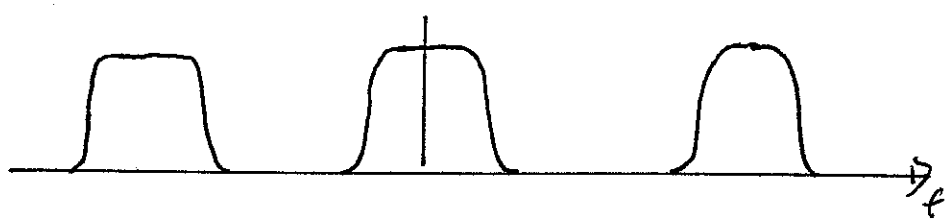
NON-LINEAR 2D DESIGN

BASIC IDEA: WORK BACKWARDS FROM DESIRED ROTATION PROFILE



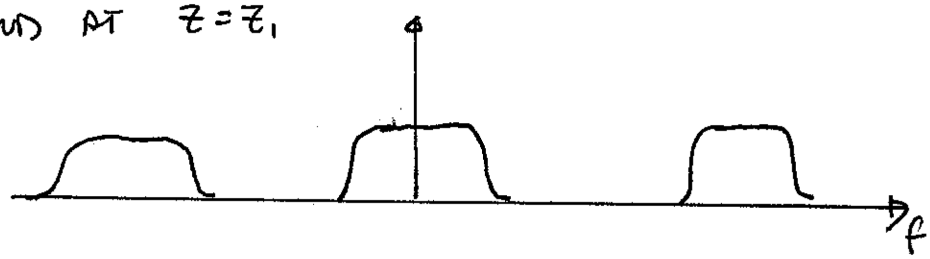
$\theta(z, f)$ OR
$\beta(z, f) = \sin(\theta(z,f)/2)$

DESIRED PROFILE AT $z = 0$



$\theta(0) = \pi$

AND AT $z = z_1$

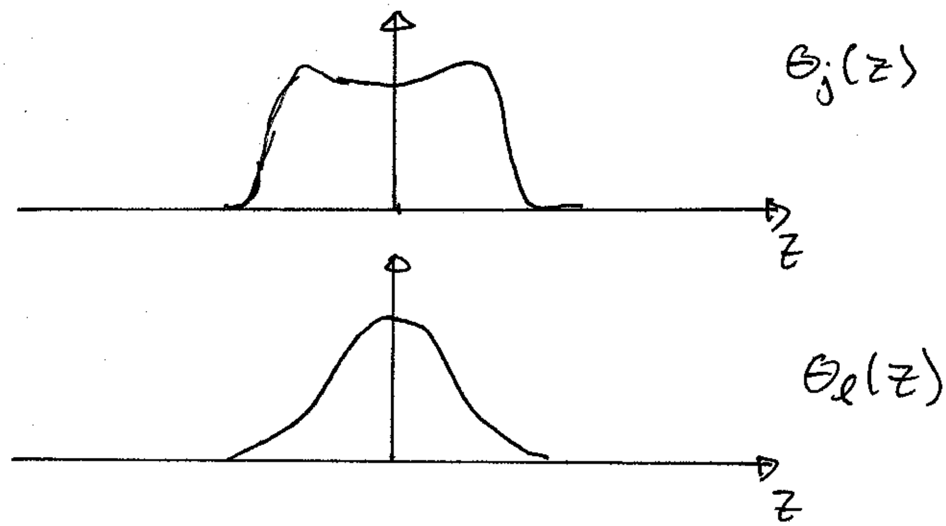


$\theta(z_1) = \pi/2$
SAME PROFILE SCALED

FOR <u>EACH</u> <u>z</u>, DESIGN A SPECTRAL PULSE FOR THE FLIP ANGLE $\theta(z)$

EACH z LOCATION HAS A DIFFERENT N-SAMPLE HARD PULSE

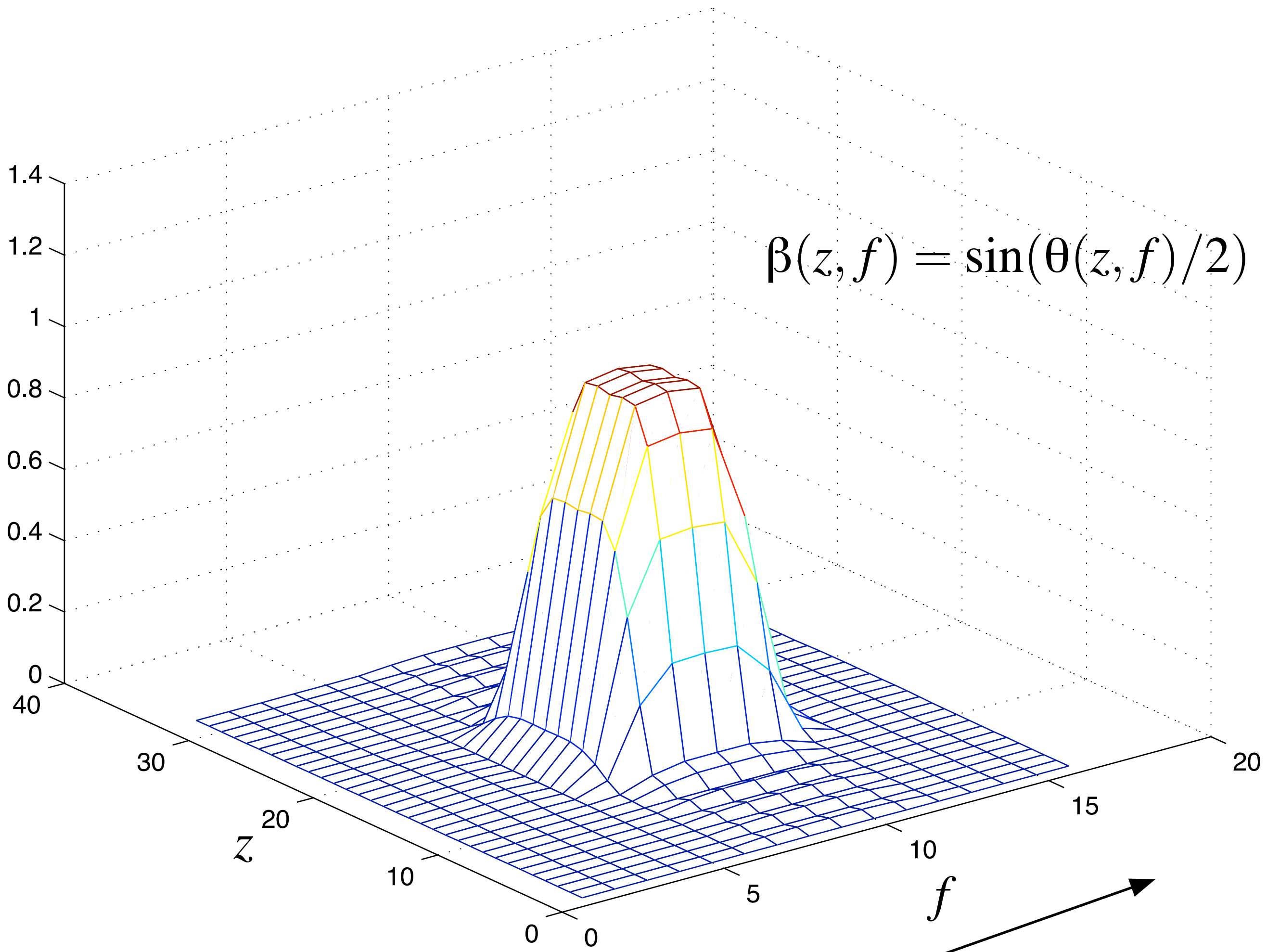
IF WE LOOK AT ONE SAMPLE OF THE HARD PULSE SEQUENCE AS A FUNCTION OF Z



EACH SUBPULSE HAS TO PRODUCE A UNIQUE ROTATION PROFILE AS A FUNCTION OF Z

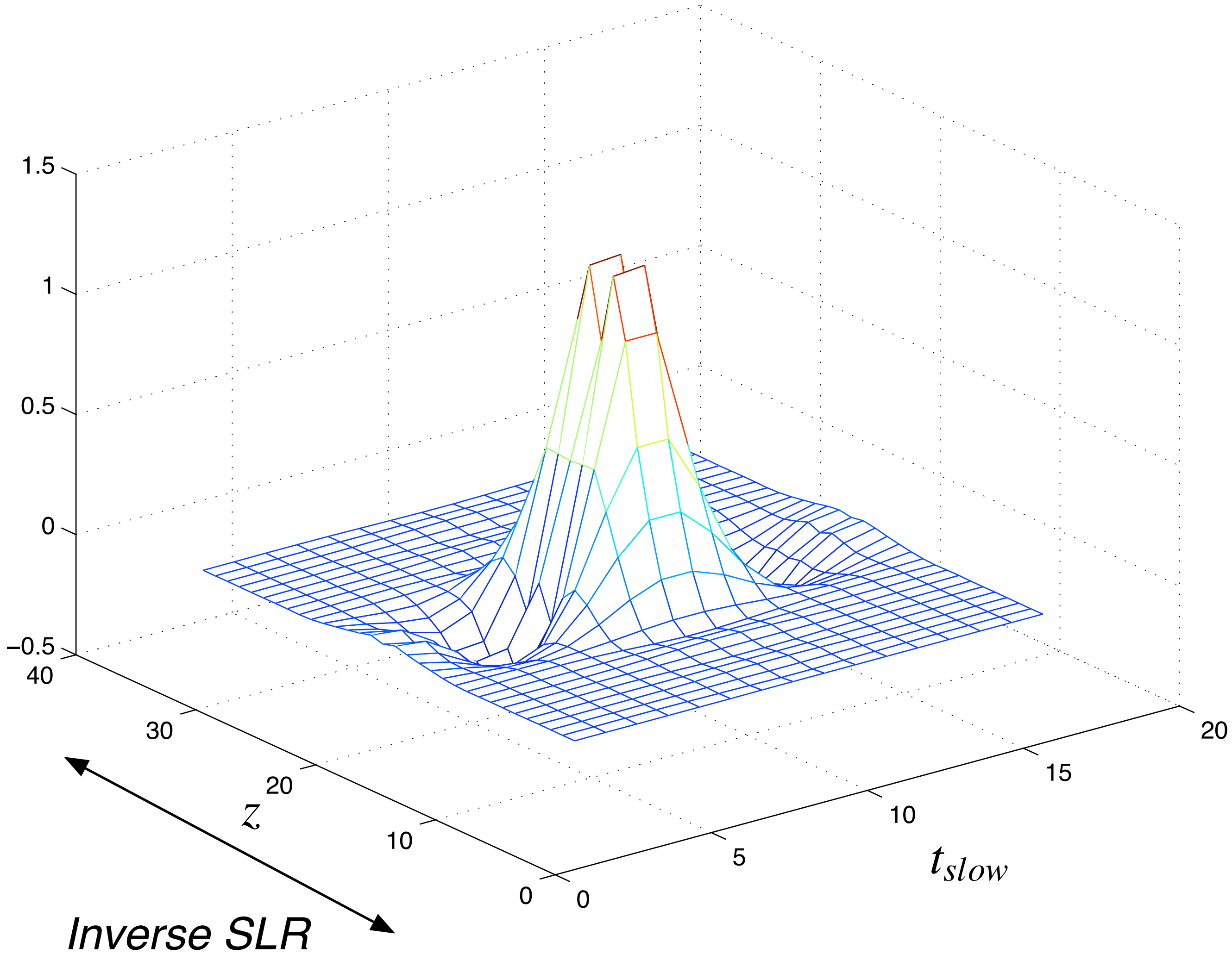
SUBPULSES ARE THEN DESIGNED EITHER BY
1) FOURIER TRANSFORM (GENERALLY SMALL ROTATION)
2) INVERSE SLR (MORE ACCURATE)

Target Function



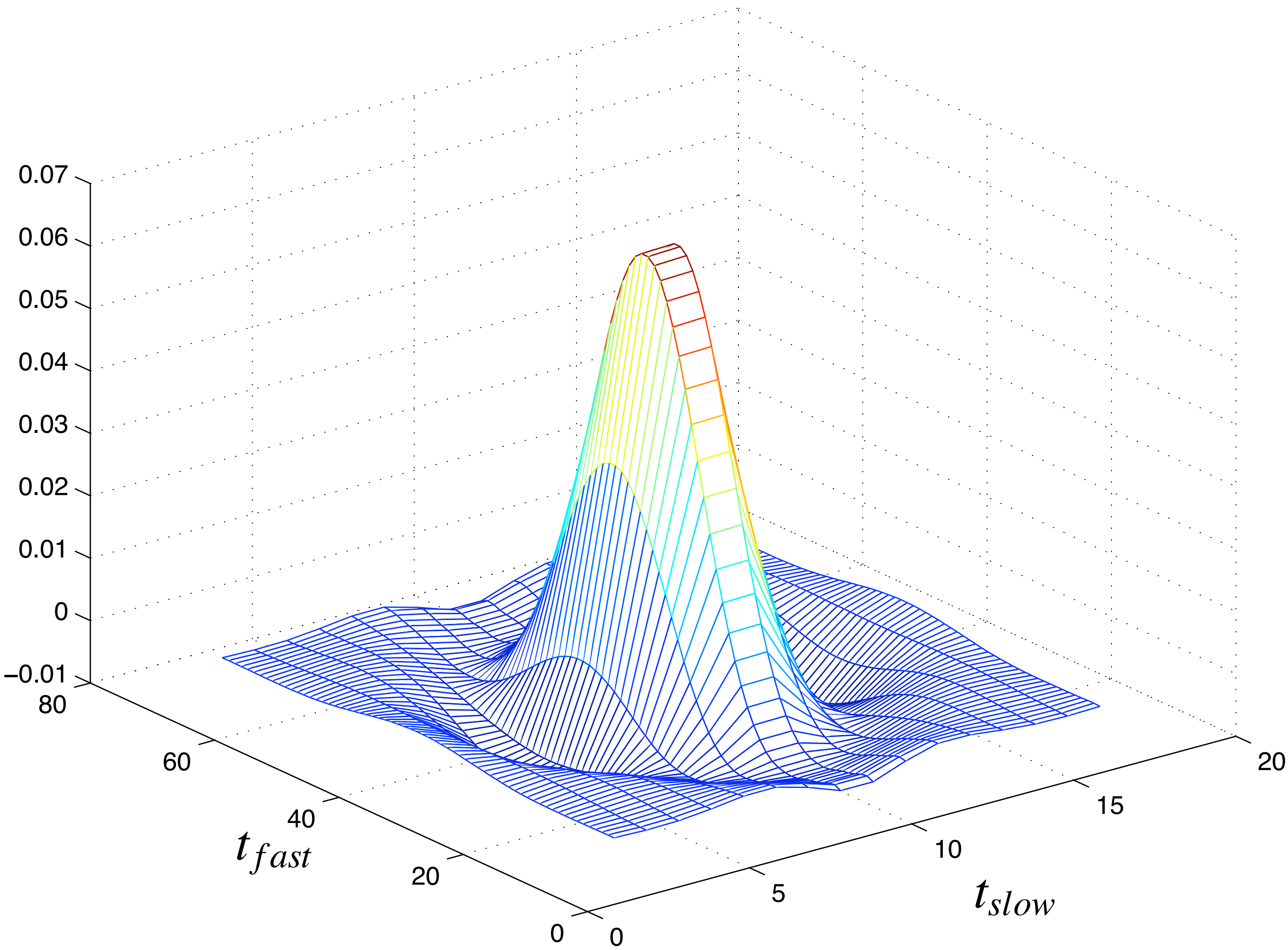
Inverse SLR
Spectral Designs

Spectral Pulses



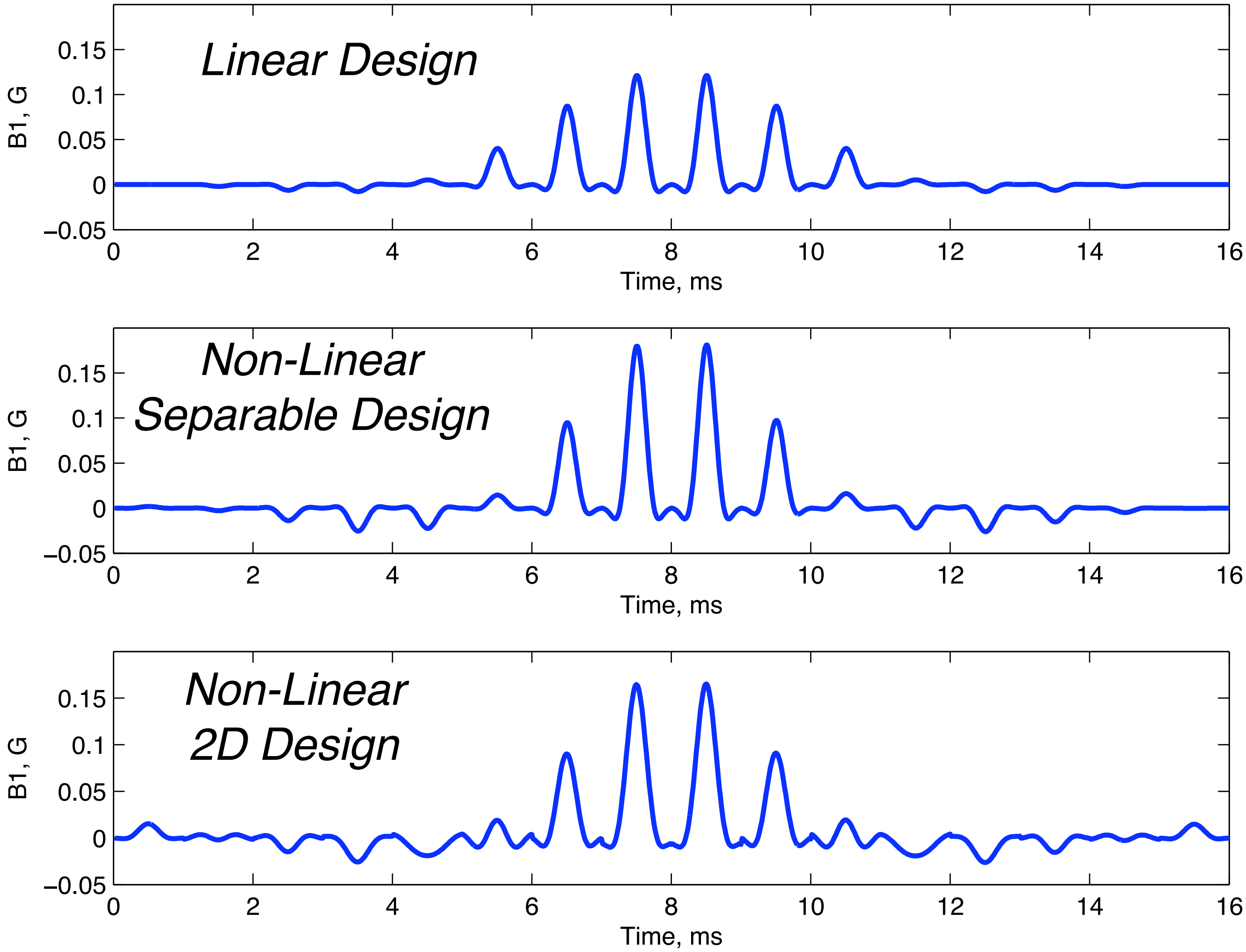
Inverse SLR Spatial Designs

Spatial Pulses

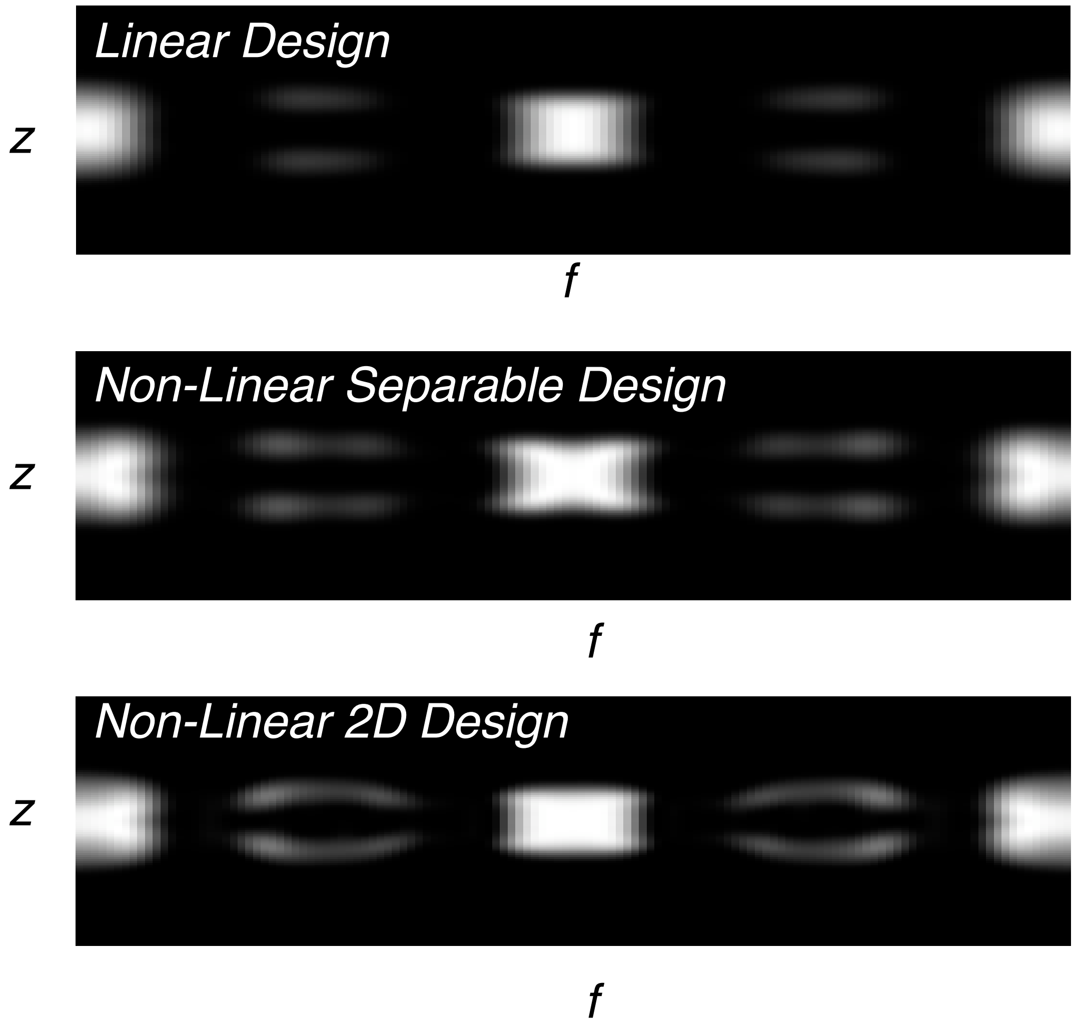


RF pulse found by tracing out surface in a raster

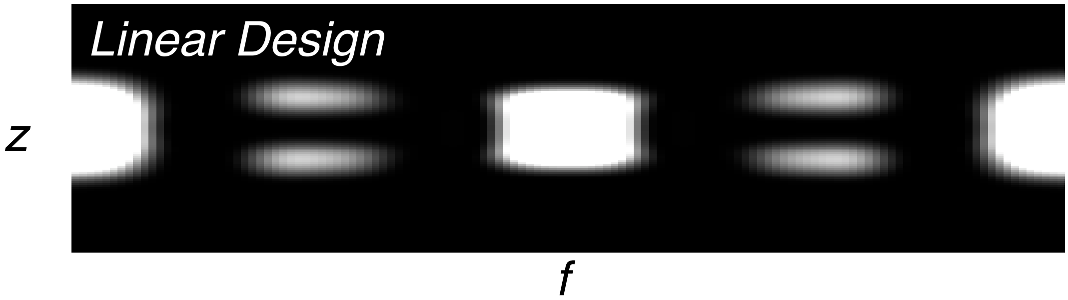
EPSE Spectral-Spatial Pulses

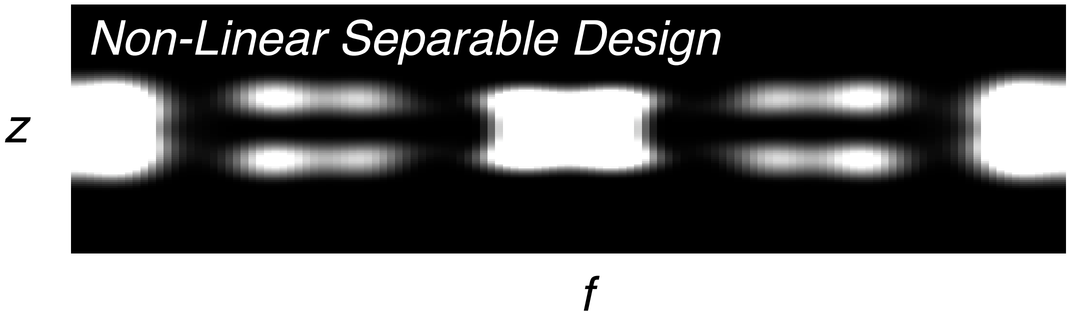


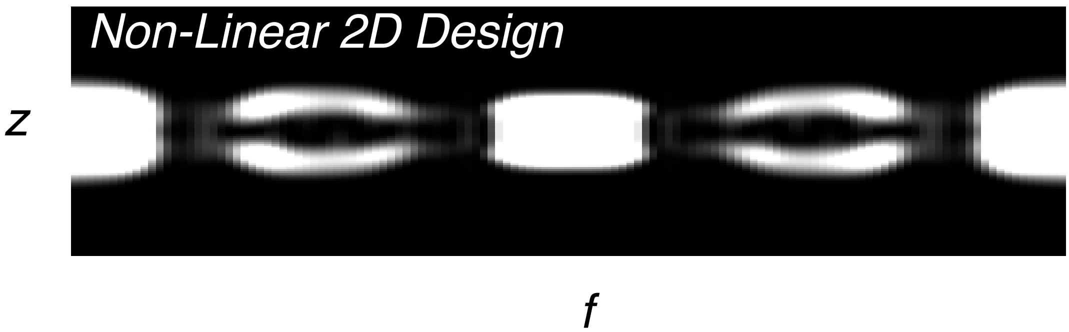
EPSE Spectral-Spatial Profiles



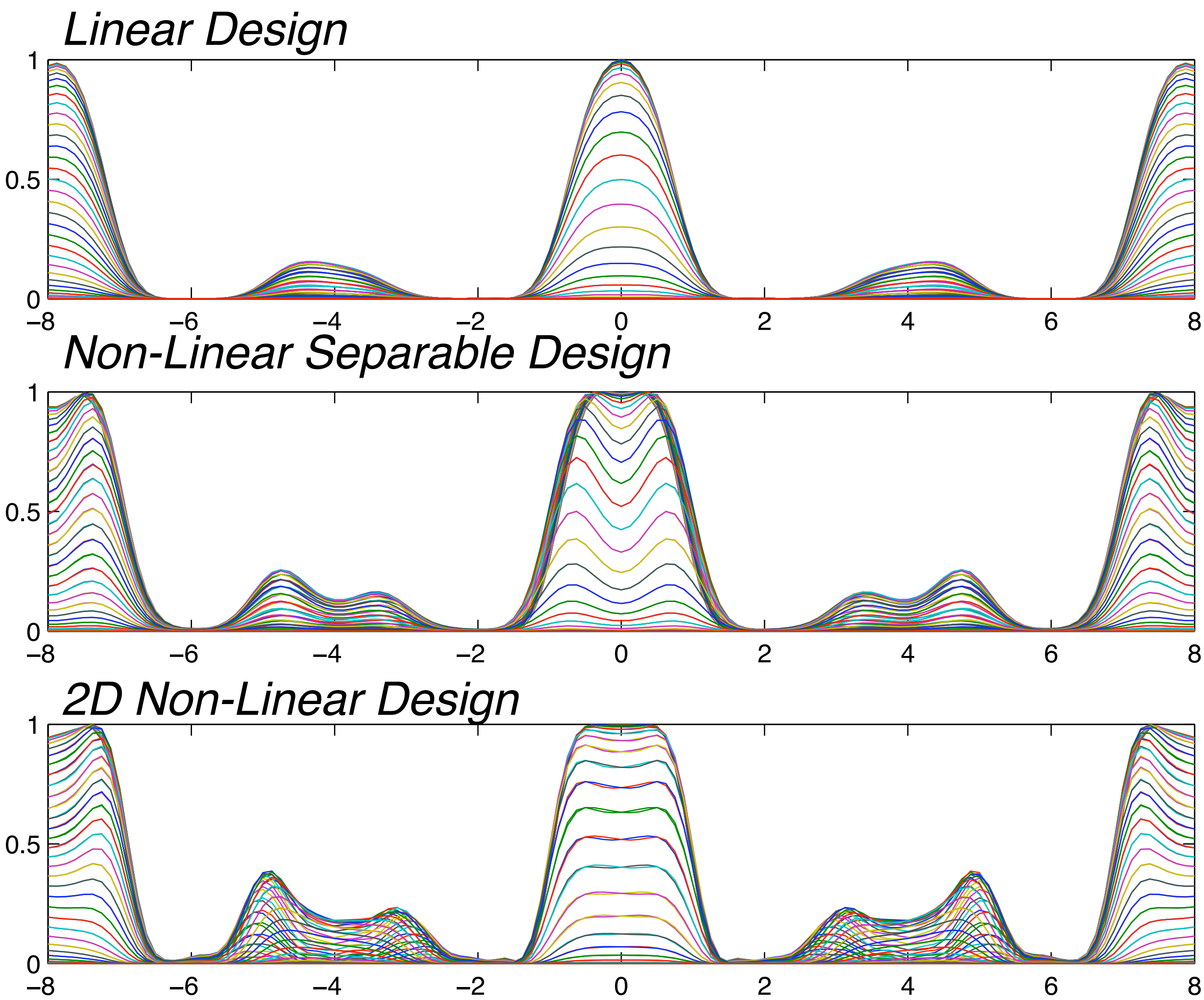
EPSE Spectral-Spatial Sidelobes







EPSE Spectral Profiles



ISSUES

1) HOW TO CHOOSE TARGET $\Theta(z,f)$

SHOULD BE "REALIZABLE"

TRANSFORM OF 2D WINDOWED SINC IS GOOD

2) FIRST STAGE (SPECTRAL PULSES) IS NON-LINEAR.

THIS CAN INTRODUCE HIGH SPATIAL FREQUENCY COMPONENTS IN SUBPULSES, WHICH MAY NOT BE LONG ENOUGH TO REPRESENT THEM.

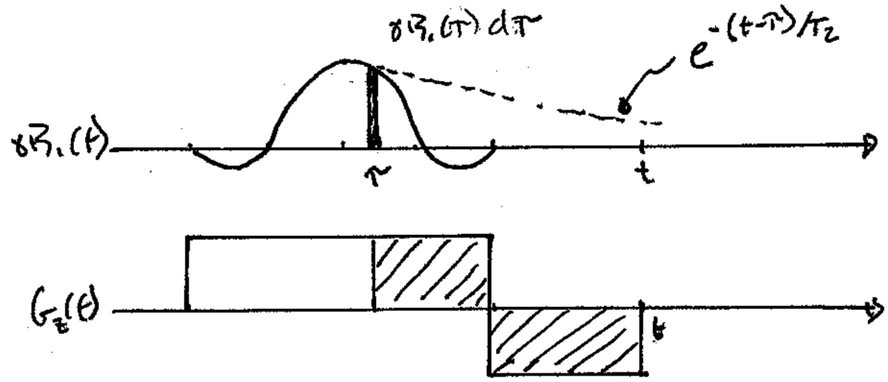
Ultra-short echo time (UTE) pulses, and short T2 contrast

- Short T2 excitation pulses
- Short T2 imaging
- Generating short T2 contrast

T2 DECAY DURING EXCITATION

SMALL-TIP-ANGLE CASE

GRAPHICAL DERIVATION



SMALL INCREMENT IN M_{xy} IS EXCITED AT TIME τ

- DECAYS BY $e^{-(t-\tau)T_2}$
- PRECESSES BY $k(\tau,t)z$ WHERE

$$k(\tau,t) = -\frac{\gamma}{2\pi}\int_{\tau}^{t} G_z(s)\,ds$$

PHASE FACTOR

$$e^{i2\pi k(\tau,t)z}$$

- INCREMENT IN M_{xy} IS

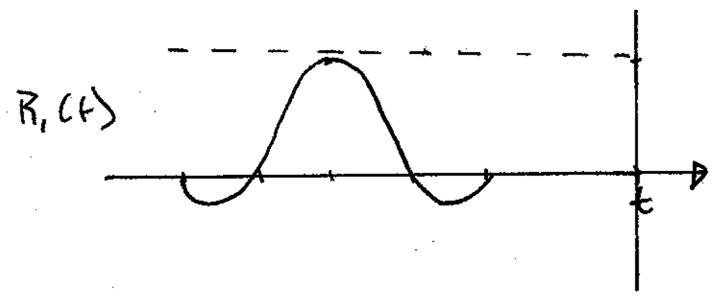
$$\Delta M_{xy} = (\gamma B_1(\tau)\Delta\tau)(iM_0)\,e^{-(t-\tau)T_2}\,e^{i2\pi k(\tau,t)z}$$

INTEGRATING

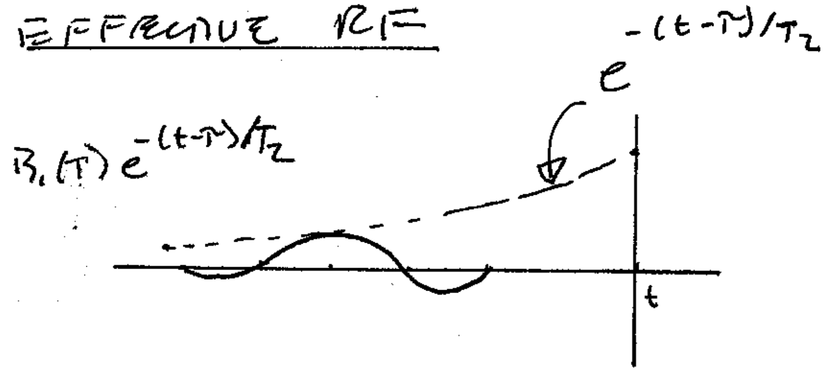
$$M_{xy}(z,t) = iM_0\int_{-\infty}^{t}\gamma B_1(\tau)\,e^{-(t-\tau)/T_2}\,e^{i2\pi k(\tau,t)z}\,d\tau$$

FOURIER TRANSFORM OF EXPONENTIALLY WEIGHTED RF

APPLIED RF



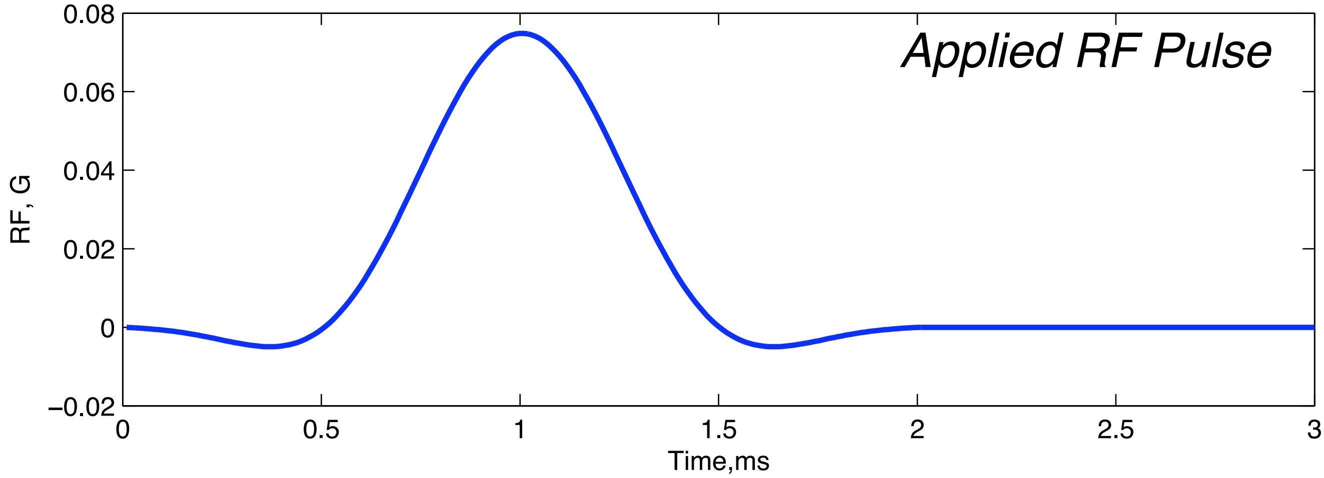
EFFECTIVE RF

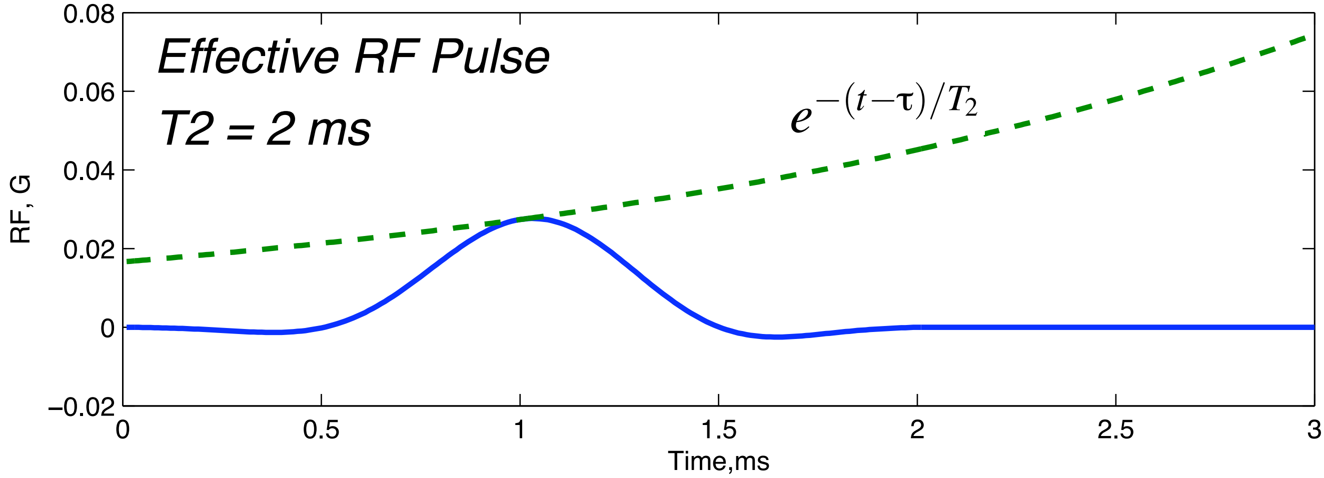


PRIMARY EFFECT:
LOSS OF SIGNAL

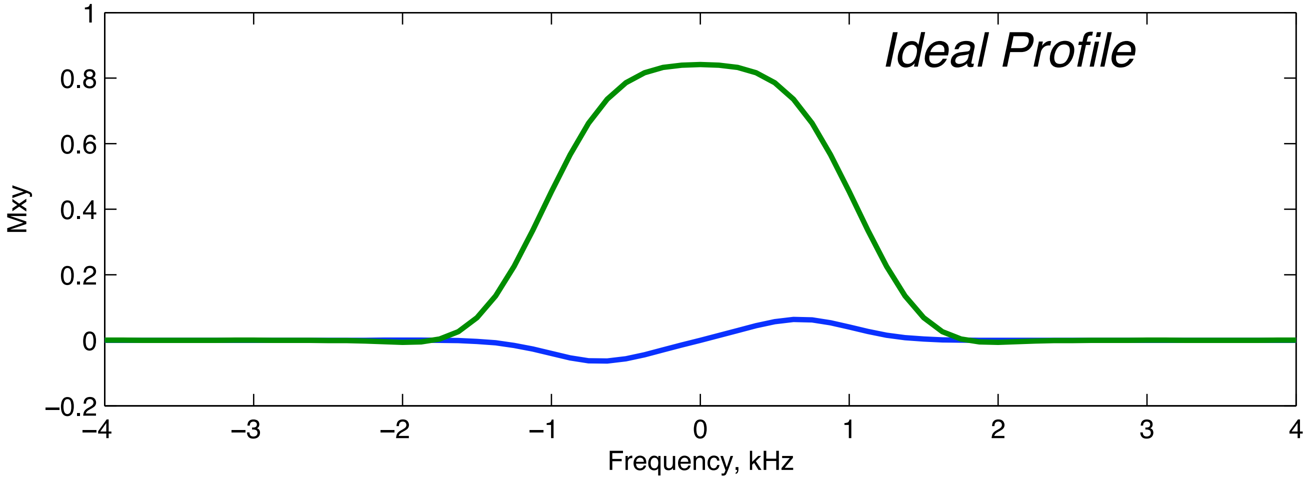
SECONDARY EFFECT:
LOSS OF SELECTIVITY

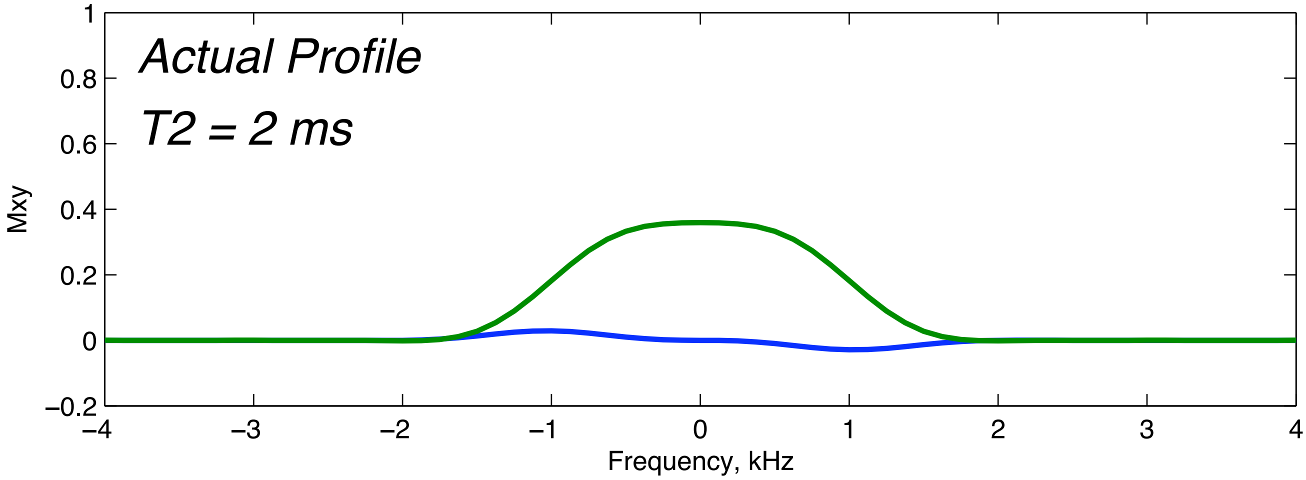
Effect of T2 on Small-Tip-Angle Excitation





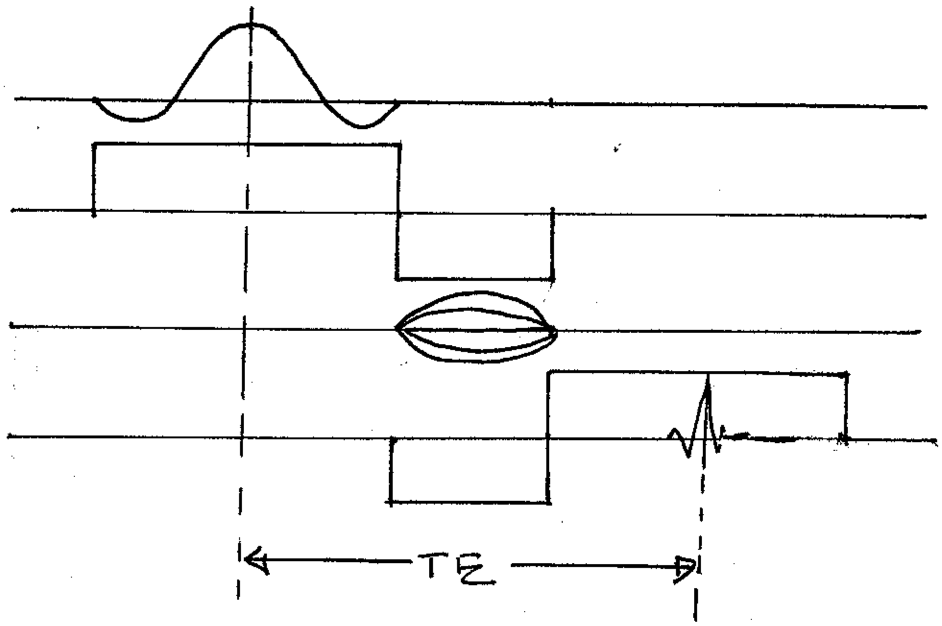
Effect of T2 on Small-Tip-Angle Excitation





ECHO TIME IN A GRADIENT RECALLED SEQUENCE

ECHO TIME IS DELAY FROM CENTER OF RF TO K-SPACE ORIGIN IN READOUT



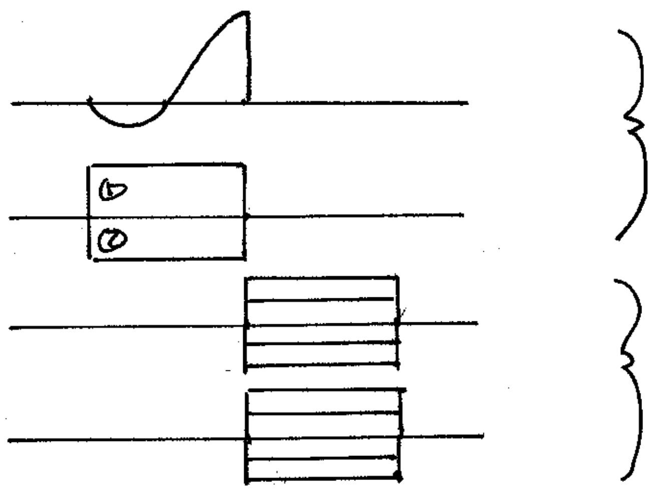
MINIMIZE T2 DECAY BY MINIMIZING TE

ELIMINATE EVERYTHING BETWEEN DASHED LINES

- HALF-PULSE EXCITATION
- RADIAL READOUT

SHORT TE PULSE SEQUENCE

IDEAL PULSE SEQUENCE

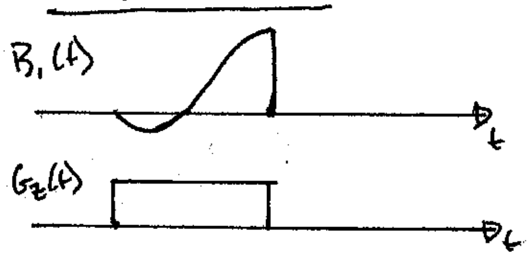


HALF-PULSE EXCITATION
TWO ACQUISITIONS COMBINED
Z-GRADIENT FLIPS

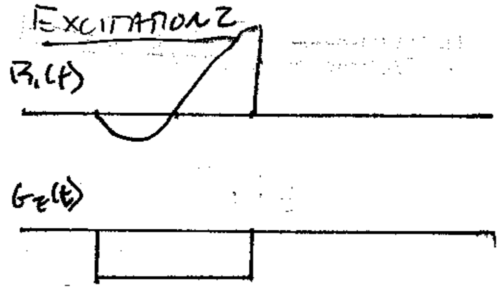
RADIAL READOUT

HALF-PULSE EXCITATION

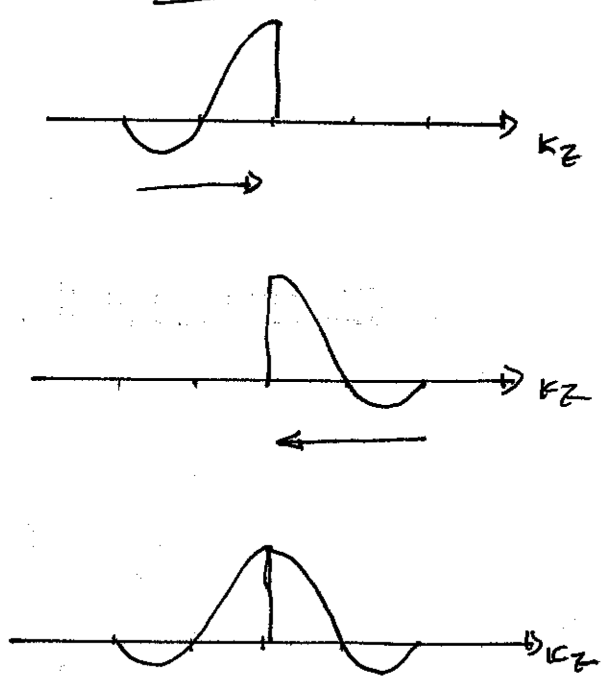
EXCITATION 1



EXCITATION 2



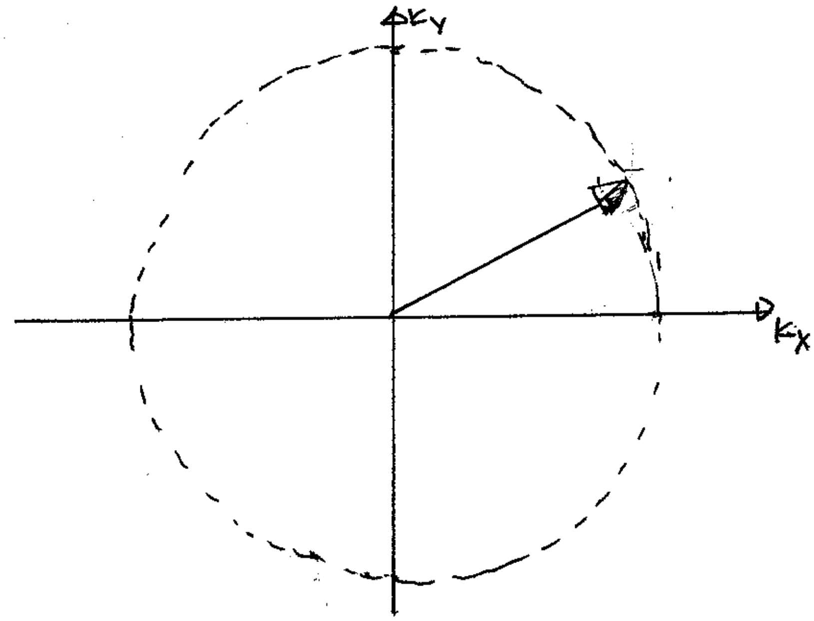
K-SPACE



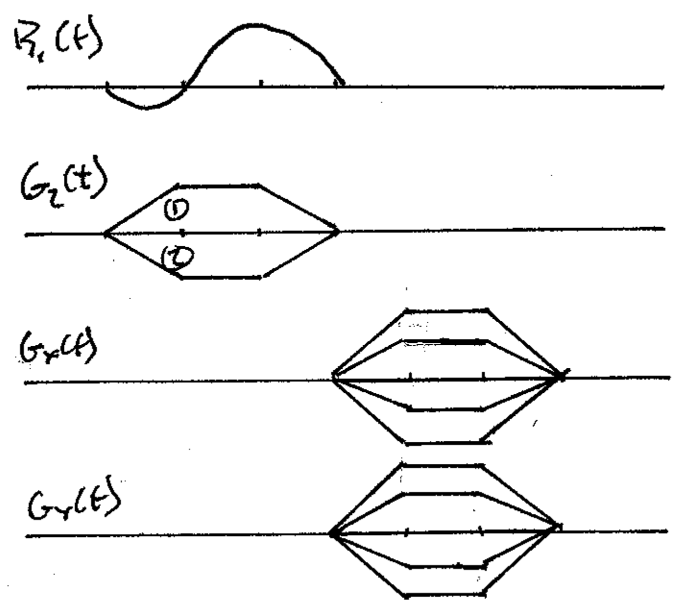
SUM OF TWO ACQUISITIONS
SAME AS CONVENTIONAL
SLICE SELECTIVE PULSE!

RADIAL READOUT

START AT K-SPACE ORIGIN, GO DIRECTLY OUT



PRACTICAL PULSE SEQUENCE



LARGE FLIP ANGLE HALF PULSES

HALF PULSES ARE BASED ON A SMALL-TIP ANGLE (FOURIER) MODEL.

WORKS WELL TO 45°, OR EVEN 60°

SIGNIFICANT DISTORTION AT 90°

SOLUTION

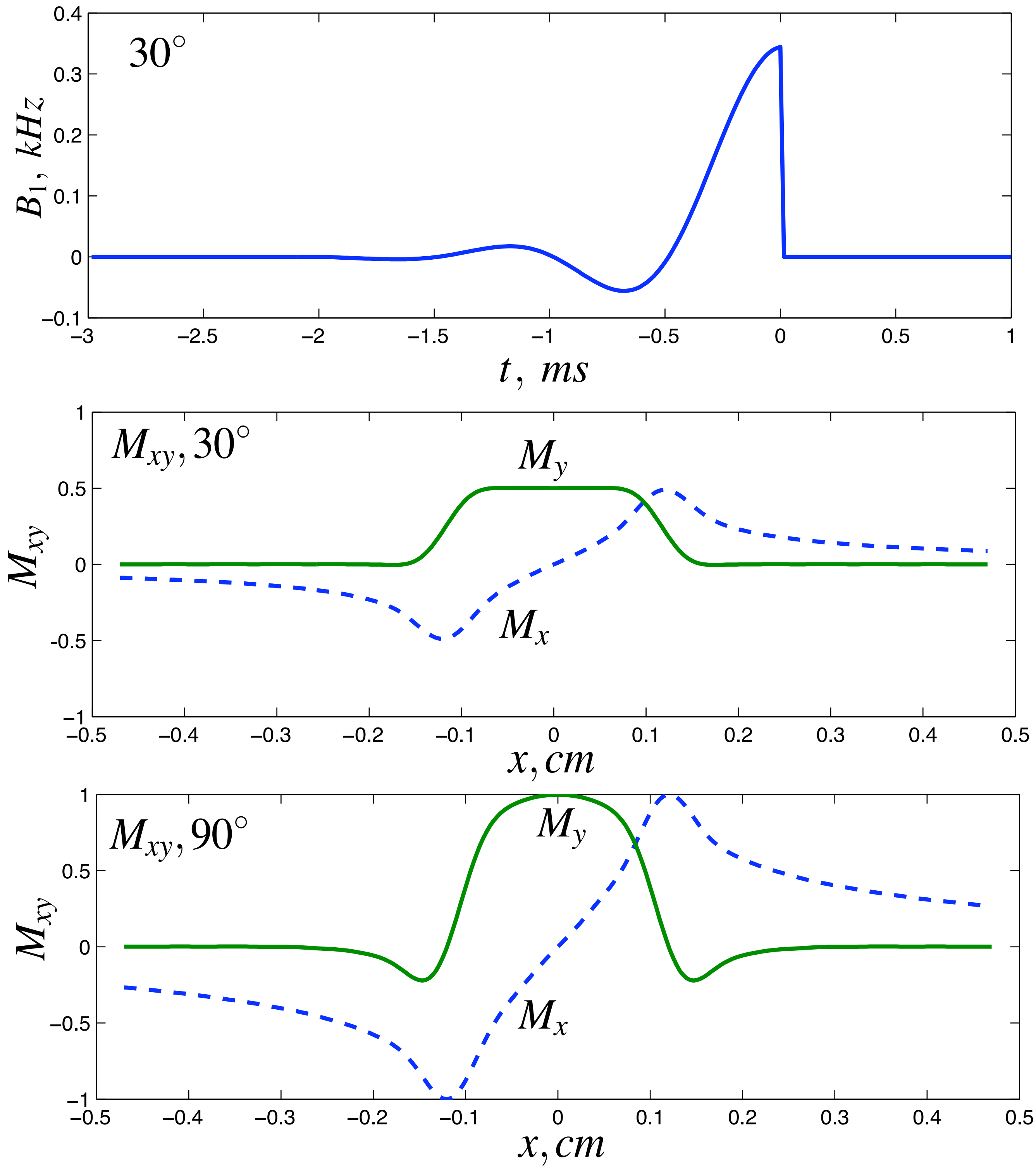
TO DESIGN A HALF PULSE FOR A FLIP ANGLE θ

1) DESIGN AN SLR PULSE FOR ANGLE 2θ

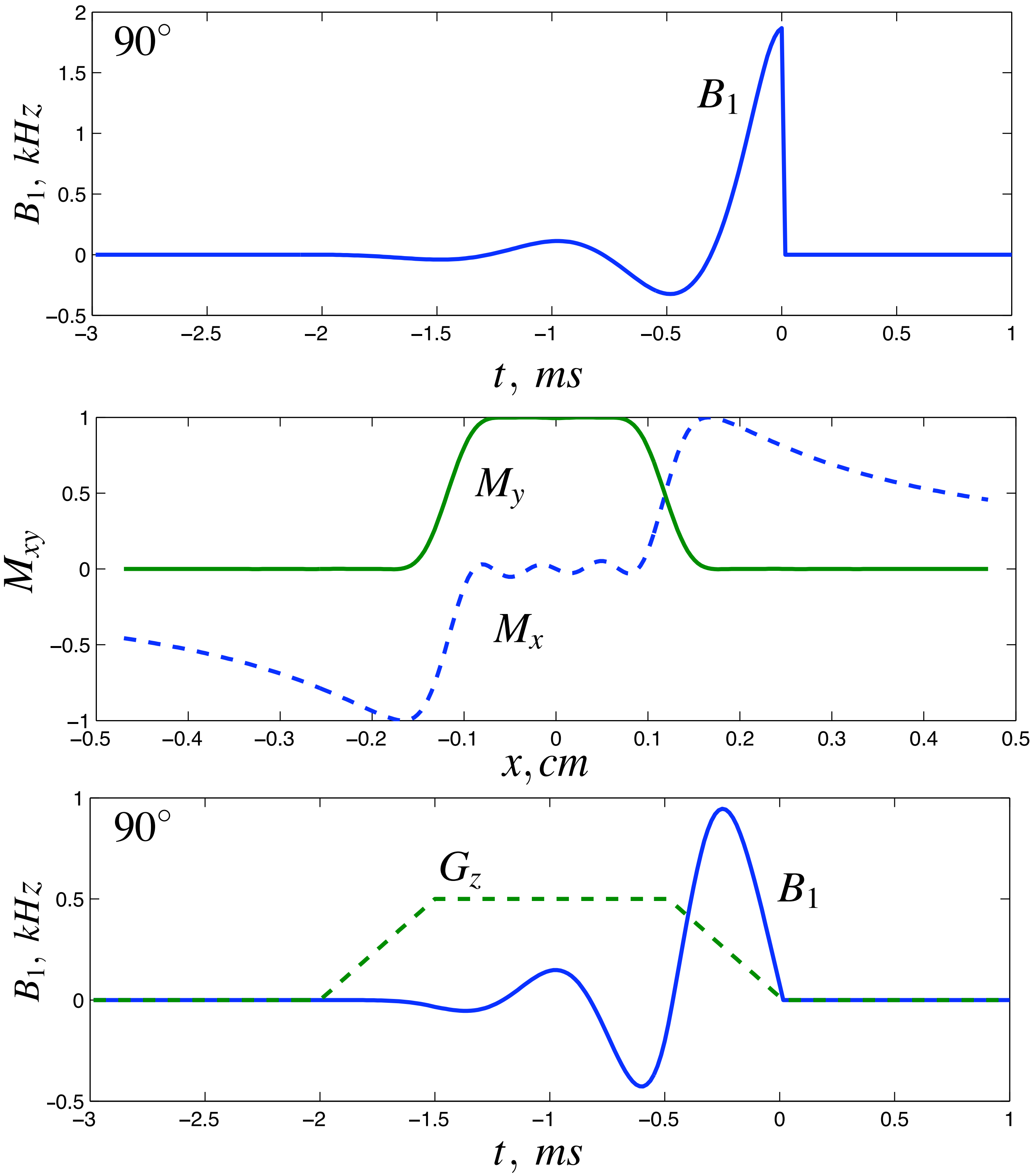
2) TAKE THE FIRST HALF.

FOR A 90° HALF PULSE, USE THE FIRST HALF OF AN SLR 180!

Large-Tip-Angle Half Pulses, Fourier Design



Large-Tip-Angle Half Pulses
SLR Design



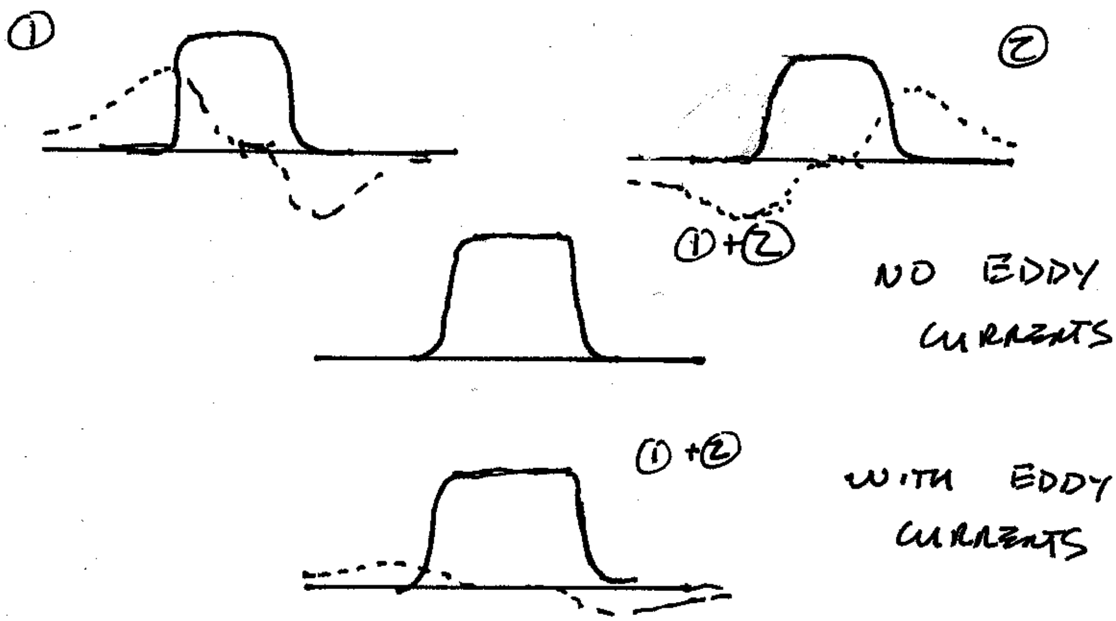
ISSUES WITH HALF PULSES

1) GRADIENT FIDELITY + EDDY CURRENT EFFECTS

WE ARE COUNTING ON

- SLICE PROFILE ADDING
- ANTI-SYMMETRIC TAILS CANCELING

BETWEEN TWO EXCITATIONS



SMALL PHASE ERRORS LEAVE RESIDUAL TAILS

THIS CAN BE A LARGE SIGNAL OVER A VOLUME

2) STEADY STATE EFFECTS

CENTER OF SLICE SEES

$\Theta_x, \Theta_x, \Theta_x$

TAIL SEES

$\phi_y, -\phi_y, \phi_y, -\phi_y$

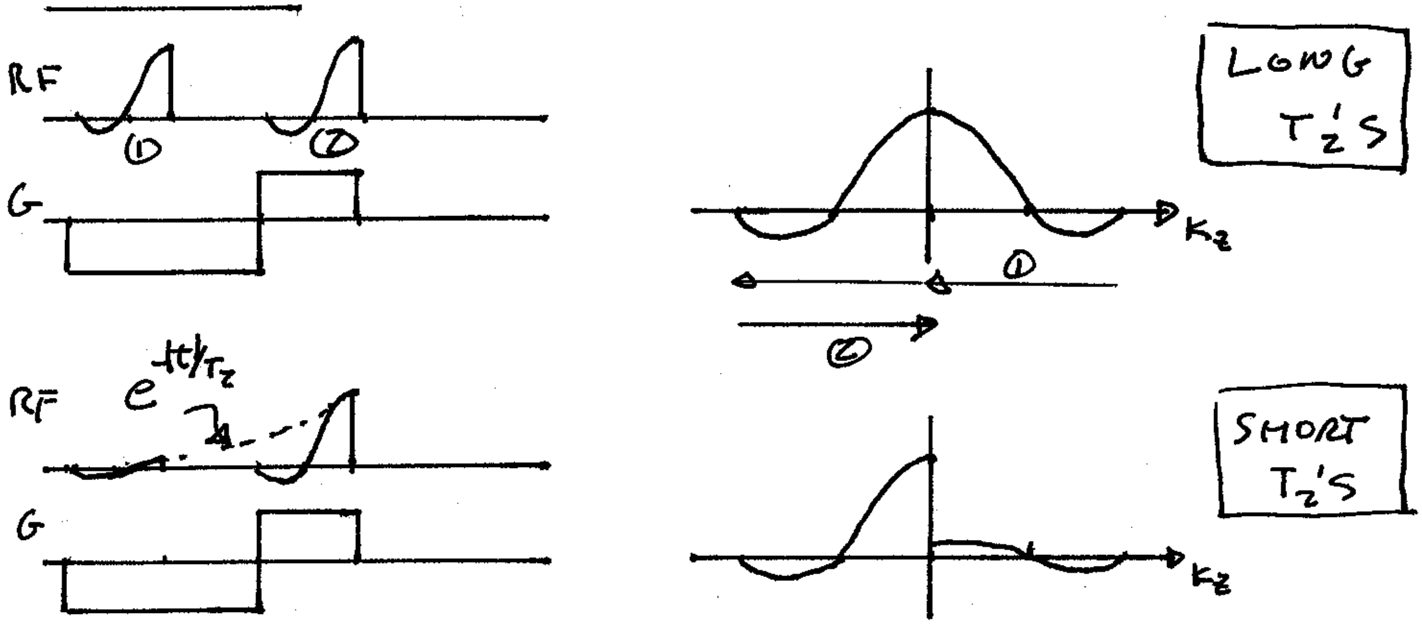
STEADY STATES CAN BE VERY DIFFERENT

DOUBLE HALF PULSES

DESIGN PULSES THAT ARE

- SLICE SELECTIVE FOR LONG T_2's (NO ASSYMETRIC TAILS)
- HALF PULSES FOR SHORT T_2'S

EXAMPLE



ADVANTAGES

NO OUT-OF-SLICE TAILS FOR LONG T_2'S

NO CANCELLATION PROBLEMS, STEADY STATE EFFECTS

IMMUNE TO EDDY CURRENT EFFECTS

EASIER TO CALIBRATE

NO EFFECTS DURING READOUT

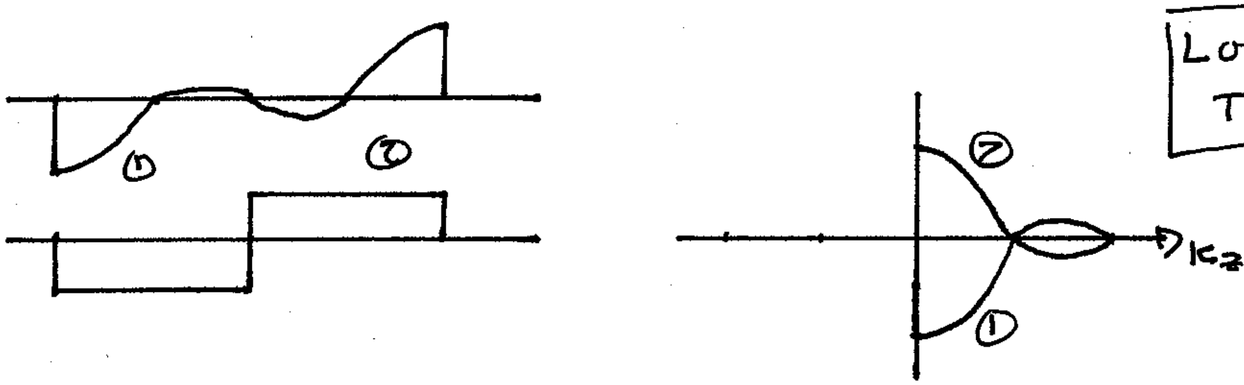
ISSUES

SPECTRAL RESPONSE

LOSS OF SHORT T_2 SIGNAL

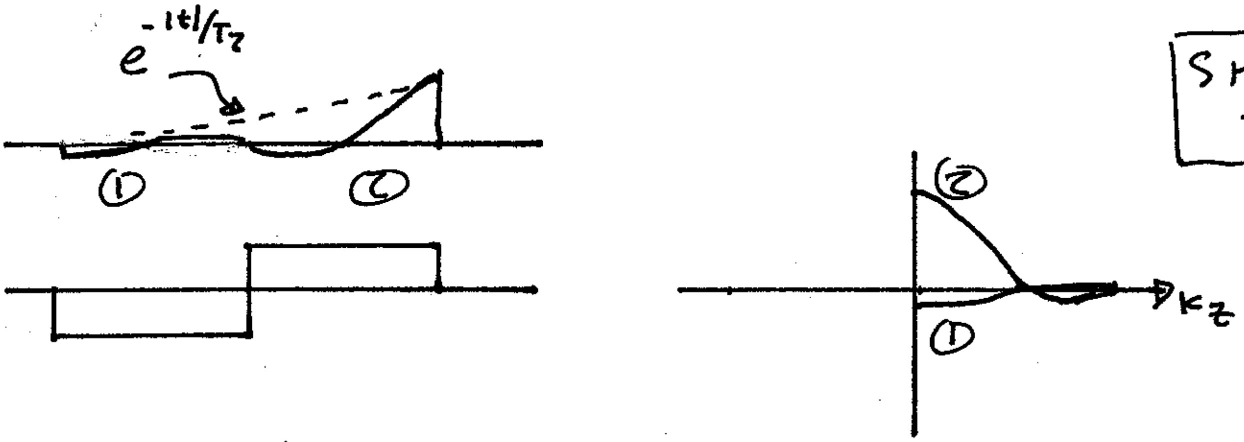
ANOTHER OPTION

LONG T_2's



LONG T_2's CANCEL.

SHORT T_2'S



SHORT T_2's SEE A HALF PULSE

PROBLEM WITH SHORT-T_2 PULSE SEQUENCE

EVERYTHING SHOWS UP

IMAGES SHOW PROTON DENSITY ONLY

HARD TO TELL WHAT HAS SHORT T_2'S

NEED SHORT-T_2 CONTRAST

SHORT-T_2 CONTRAST OPTIONS

1) COLLECT SECOND IMAGE WITH TE2, COMPUTE DIFFERENCE IMAGE.

$$I_{ST2} = I_{TE1} - I_{TE2}$$

SHOULD SHOW ONLY SHORT-T2 COMPONENTS.

PROBLEMS:

- SECOND IMAGE ONLY ADDS NOISE TO SHORT-T2 VOXELS
- INHOMOGENEITY IN A VOXEL LOOKS LIKE SHORT-T_2

2) EXPLOIT T_1 DIFFERENCES

TISSUES WITH SHORT T_2'S TEND TO HAVE SHORT T_1'S (NOT LIKE SOLIDS)

RF SPOILED SHORT TR SEQUENCES WILL SHOW SHORT T1'S, EMPHASIZE SHORT T2'S

PROBLEMS:

- STILL NOT T_2 SELECTIVE UNLESS COMBINED WITH (1).

3) EXPLOIT EXCITATION DIFFERENCES

T2 EFFECTS OUR ABILITY TO EXCITE OR INVERT MAGNETIZATION

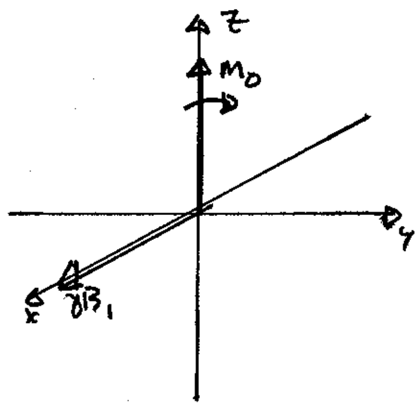
USE PREPULSES TO ESTABLISH SHORT-T_2 CONTRAST.

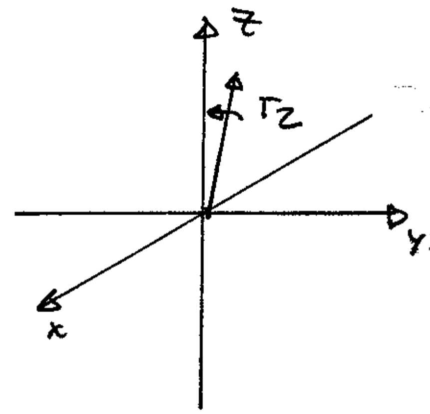
DESIGN T_2 SELECTIVE EXCITATIONS

LONG T_2 SUPPRESSION PULSES

BASIC IDEA: EXCITATION AND RELAXATION ARE COMPETING PROCESSES

EXCITATION CREATES MAGNETIZATION (m_{xy}) - AND - RELAXATION DESTROYS MAGNETIZATION





BOTH ARE EFFECTIVELY ROTATION RATES FOR SMALL-TIP-ANGLE CASE

IF $\gamma B_1 \gg T_2$

MAGNETIZATION IS ROTATED AWAY FROM +z

IF $\gamma B_1 \ll T_2$

NO TRANSVERSE MAGNETIZATION IS CREATED

$\underline{M}$ STAYS UNTOUCHED ALONG +z

LONG T_2 SUPPRESSION PULSE

LONG, LOW AMPLITUDE 90° PULSE

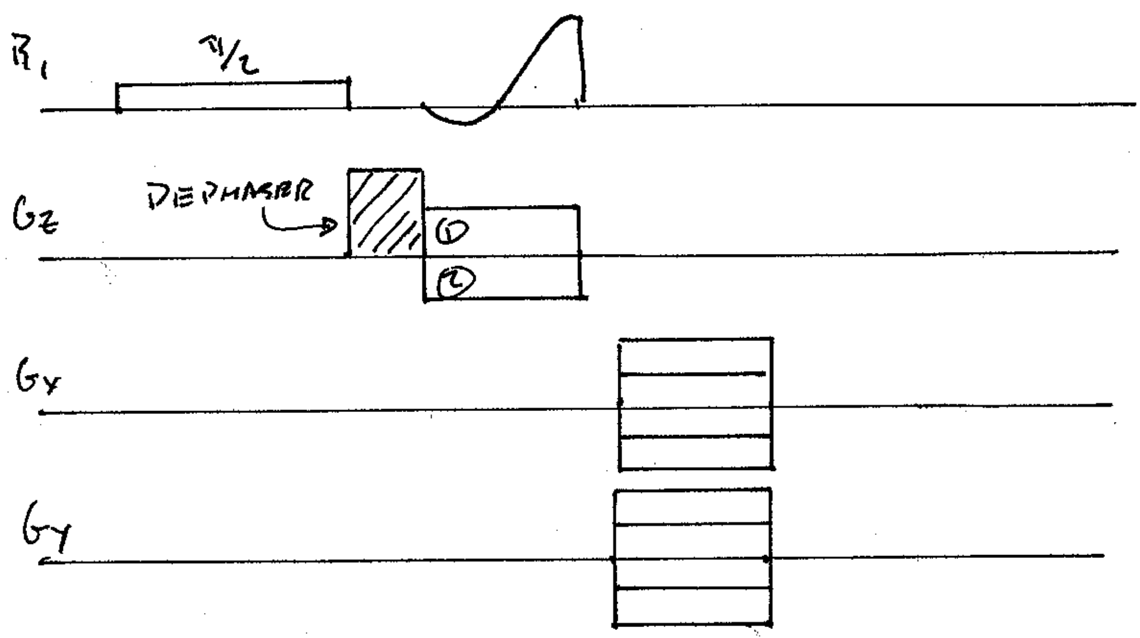
IF T IS PULSE LENGTH

T_2'S $<< T$ ARE UNEXCITED

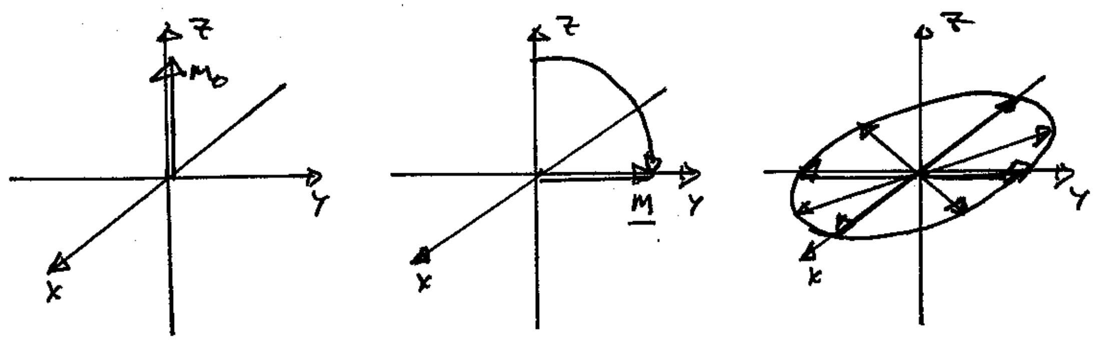
T_2'S $>> T$ ARE COMPLETELY EXCITED

EXCITED MAGNETIZATION IS DEPHASED

REMAINING MAGNETIZATION IS IMAGED WITH SHORT-T_2 PULSE SEQUENCE

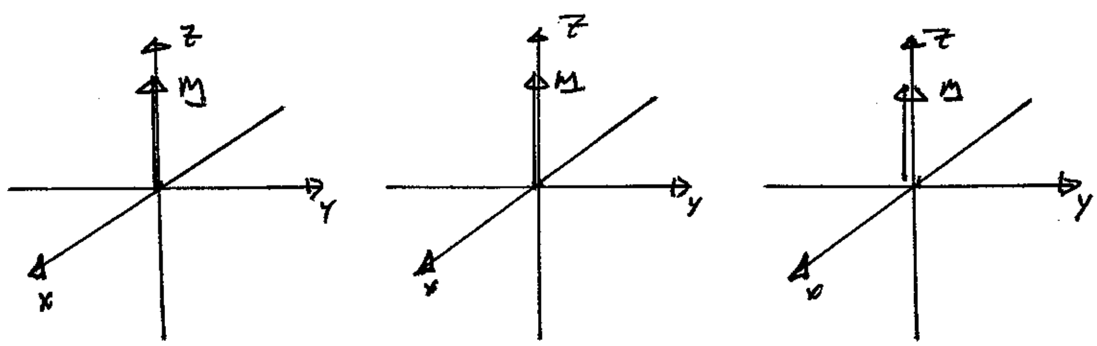


LONG-T_2 SPECIES



EQUILIBRIUM EXCITATION DEPHASING

SHORT-T_2 SPECIES



EQUILIBRIUM EXCITATION DEPHASING

LONG-T_2 SPECIES ARE EXCITED AND DEPHASED

SHORT-T_2 SPECIES ARE NOT EXCITED AT ALL.

LONG-T_2 SUPPRESSION RESPONSE

RECTANGULAR $\pi/2$ PULSE

$$\begin{pmatrix} \dot{m}_x \\ \dot{m}_y \\ \dot{m}_z \end{pmatrix} = \begin{pmatrix} -1/T_2 & 0 & 0 \\ 0 & -1/T_2 & \gamma B_1 \\ 0 & -\gamma B_1 & 0 \end{pmatrix} \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix}$$

LAST TWO EQUATIONS DECOUPLE

$$\begin{pmatrix} \dot{m}_y \\ \dot{m}_z \end{pmatrix} = \begin{pmatrix} -1/T_2 & \gamma B_1 \\ -\gamma B_1 & 0 \end{pmatrix} \begin{pmatrix} m_y \\ m_z \end{pmatrix}$$

WRITE AS A SINGLE EQUATION IN m_z

$$\ddot{m}_z + \frac{1}{T_2}\dot{m}_z + (\gamma B_1)^2 m_z = 0$$

LINEAR CONSTANT COEFFICIENT ODE. ROOTS ARE

$$\lambda_{1,2} = -\frac{1}{2T_2} \pm \sqrt{\left(\frac{1}{2T_2}\right)^2 - (\gamma B_1)^2}$$

SOLUTION FOR m_z IS

$$m_z(t) = \frac{1}{\lambda_1 - \lambda_2}\left[\left(\lambda_1 + \frac{1}{T_2}\right)e^{\lambda_1 t} - \left(\lambda_2 + \frac{1}{T_2}\right)e^{\lambda_2 t}\right]$$

FOR A $\pi/2$ PULSE, IF THE PULSE LENGTH IS T,

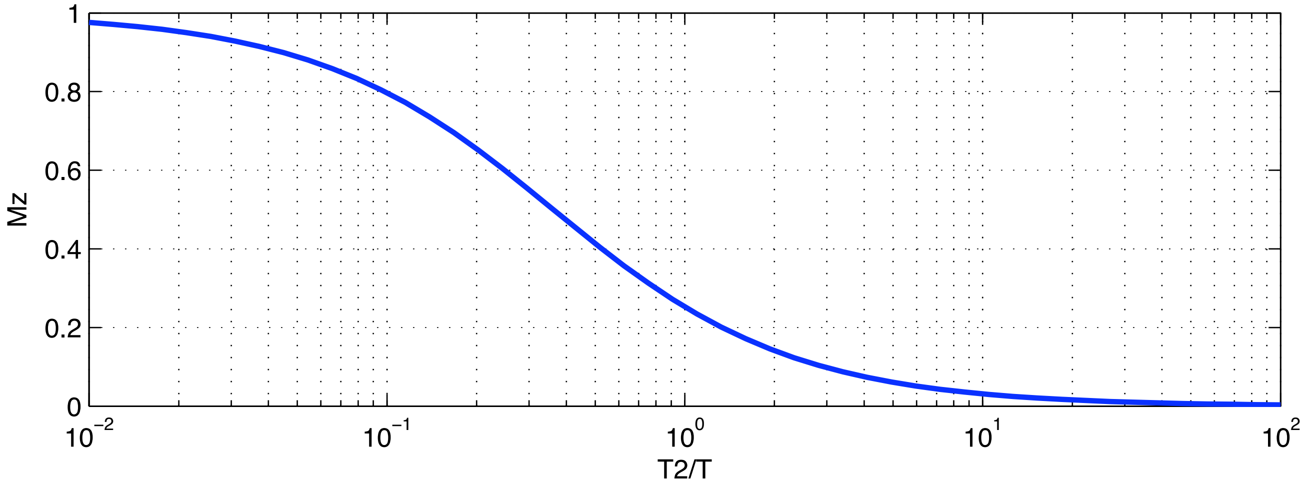
$$\gamma B_1 = \frac{\pi/2}{T} = \frac{\pi}{2T}$$

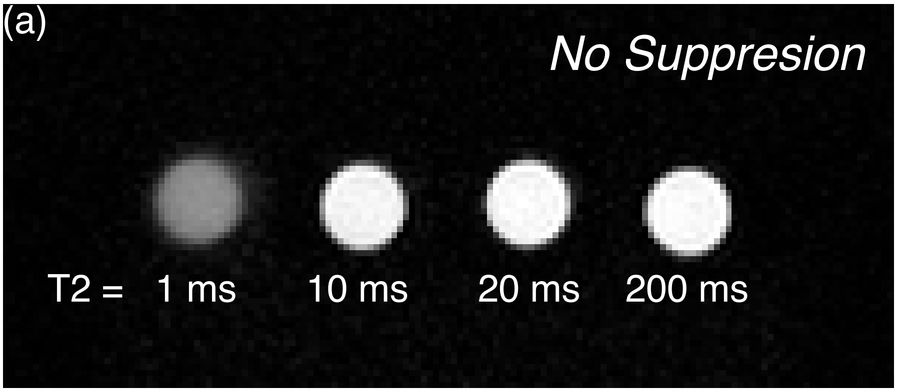
SUBSTITUTING FOR $t=T$, AND γB_1, AND SIMPLIFYING CONSIDERABLY

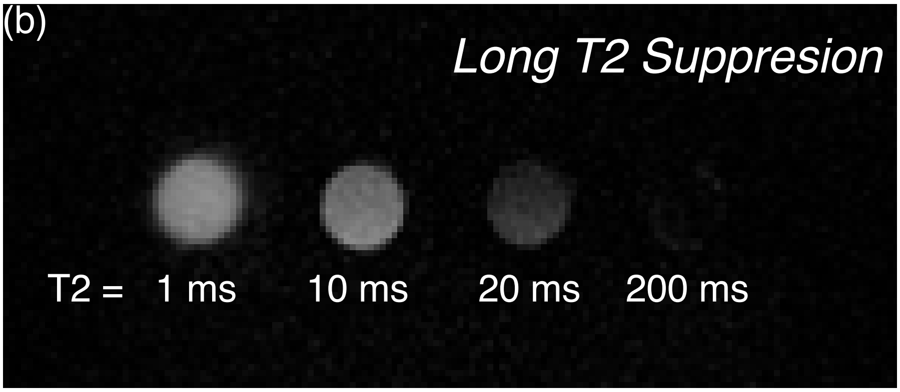
$$M_z(T) = e^{-\frac{\pi}{2}\left(\frac{T}{\pi T_2}\right)}\left\{\frac{(T/\pi T_2)}{\sqrt{\left(\frac{T}{\pi T_2}\right)^2-1}}\sinh\left(\frac{\pi}{2}\sqrt{\left(\frac{T}{\pi T_2}\right)^2-1}\right) + \cosh\left(\frac{\pi}{2}\sqrt{\left(\frac{T}{\pi T_2}\right)^2-1}\right)\right\}$$

THIS IS A FUNCTION OF T_2/T.

Long-T2 Suppression







FOR A GIVEN PULSE LENGTH T

- T_2'S < T ARE PRESERVED
- T_2'S > T ARE SUPPRESSED

IF WE WANT TO PRESERVE T_2'S OF 2ms WITH 80% EFFICIENCY, WE NEED A 10×2ms = 20ms PULSE!

PROBLEM: A 20ms RECTANGULAR PULSE HAS A BANDWIDTH OF

$$\frac{1}{20ms} = \underline{50\,HZ}$$

THIS IS MUCH LESS THAN B_0 INHOMOGENEITY EXPECTED OVER MUCH OF BODY (@ 1.5T)

CAN I USE ANOTHER WAVEFORM TO BROADEN THE BANDWIDTH?

UNDER REASONABLE ASSUMPTIONS, WE CAN SHOW:

$$M_Z(T) \simeq M_0 \left(1 - T_2 \int_{-\infty}^{T} (\gamma B_1(\tau))^2 \, d\tau \right)$$

THIS MEANS THAT THE DECREASE IN M_Z IS DUE ONLY TO

1) T_2

2) RF POWER

T_2 IS FIXED, SO OUR ONLY VARIABLE IS RF POWER.

RESULT:

WIDER BANDWIDTH SUPPRESSION PULSE WILL ALSO SUPPRESS SHORT T_2's MORE, DUE TO PARSEVAL'S THEOREM.

IF

$$\omega_1(t) = \gamma B_1(t)$$
$$\Omega_1(f) = \mathcal{F}\{\omega_1(t)\}$$

THEN

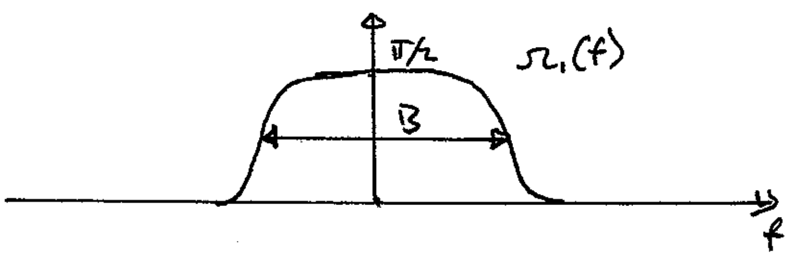
$$\int_{-\infty}^{\infty} |\omega_1(t)|^2 \, dt = \int_{-\infty}^{\infty} |\Omega_1(f)|^2 \, df$$

IN ORDER TO MINIMIZE SUPPRESSION OF SHORT T_2'S, WHICH IS

$$M_z = 1 - \underbrace{T_2 \int_0^T |\omega_1(t)|^2 dt}_{\text{MINIMIZE THIS}}$$

$$= 1 - T_2 \int_{-\infty}^{\infty} |\Omega_1(f)|^2 df$$

FOR LONG-T_2'S THE EXCITATION PROFILE IS $\mathcal{F}\{\gamma B_1(t)\} = \Omega_1(f)$ AND THIS SHOULD BE $\pi/2$ IN THE SUPPRESSION BAND



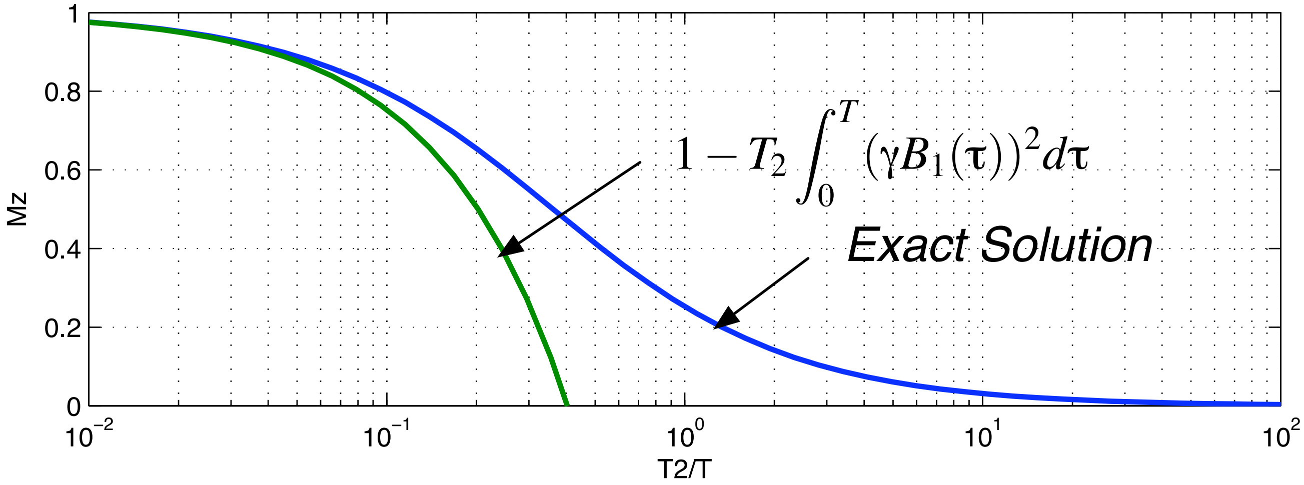
BANDWIDTH B

THE SUPPRESSION IS THEN

$$M_z \simeq 1 - T_2 \left(\frac{\pi}{2}\right)^2 B$$

NEED TO MINIMIZE B.

Approximate vs Exact Solution Rectangular Suppression Pulse



Pulse pairs, and self-refocusing pulses

- Self-refocusing pulses (true spin-echo pulses)
- Spin echo pulse pairs

SELF REFOCUSING PULSES

GOAL: PULSE THAT PRODUCES A SPIN ECHO
AT THE END, OR AFTER, THE END OF THE PULSE

TRANSVERSE MAGNETIZATION IS

$$m_{xy} = 2\alpha^* \beta$$
$$= 2 A^*(z) B(z) \qquad z = e^{i\gamma G x T}$$

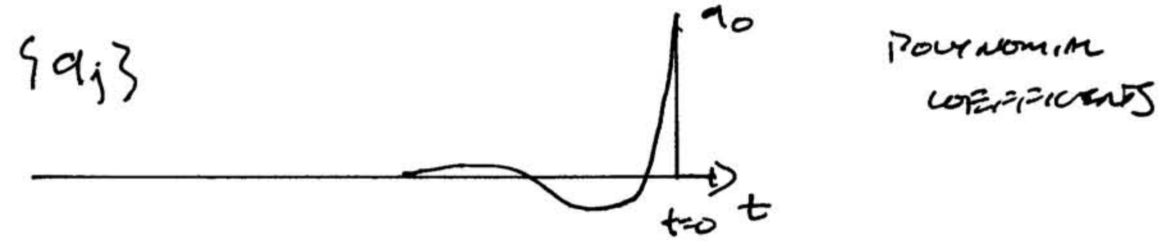
WE CAN DELAY THE ECHO BY USING A MINIMUM PHASE B(Z), MINIMUM PHASE RF

THIS STILL HAS SOME PHASE, AND NEEDS TO BE REFOCUSED

IN ORDER TO COMPENSATE THIS PHASE, WE NEED TO ADD PHASE TO A(Z).

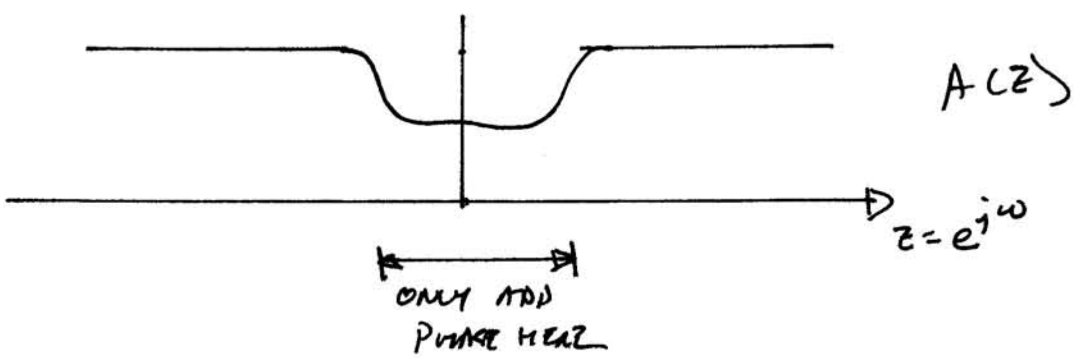
$$A_p(z) = P(z) A(z)$$

PHASE MUST BE ADDED CAREFULLY SINCE RF POWER IS PROPORTIONAL TO $1-q_0$



WANT $\{q_j\}$ SEQUENCE TO BE CLOSE TO AN IMPULSE

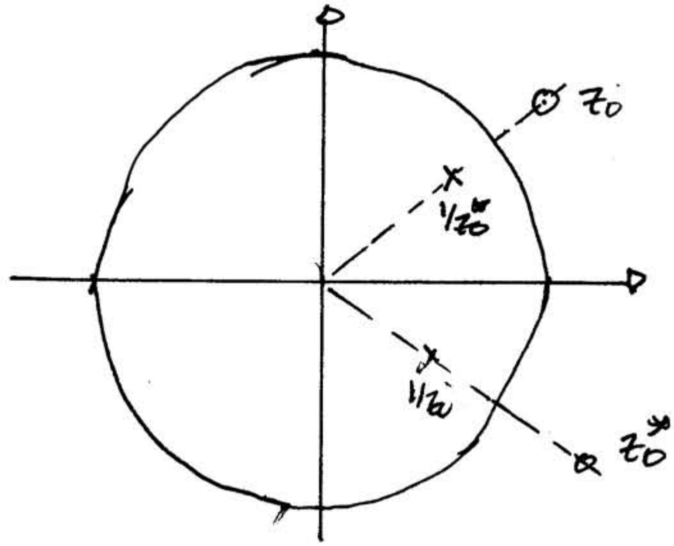
ONLY ADD PHASE OVER PASSBAND



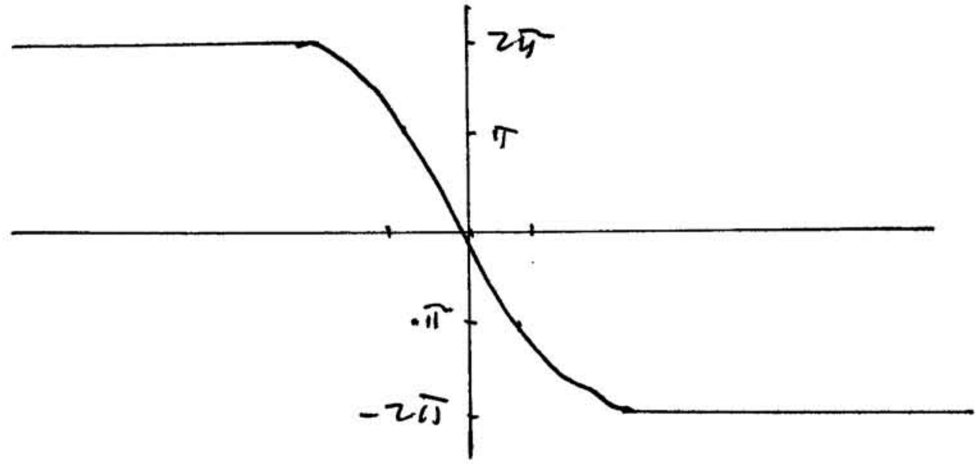
ONE OPTION, ALLPASS FUNCTION

$$P(z) = \frac{1}{|z_0|^2} \frac{(z-z_0)(z-z_0^*)}{(z-1/z_0)(z-1/z_0^*)}$$

POLE / ZERO PLACEMENT



PHASE PROFILE



4π PHASE SHIFT ACROSS PASSBAND

$2\ \Delta T$ SHIFT IN TIME, $\Delta T = \frac{T}{(TBW)}$

POWER $\sim 1 - |z_0|$

CONSTRAIN $|z_0|$, OPTIMIZE ANGLE
(LEAST SQUARES SOLN)

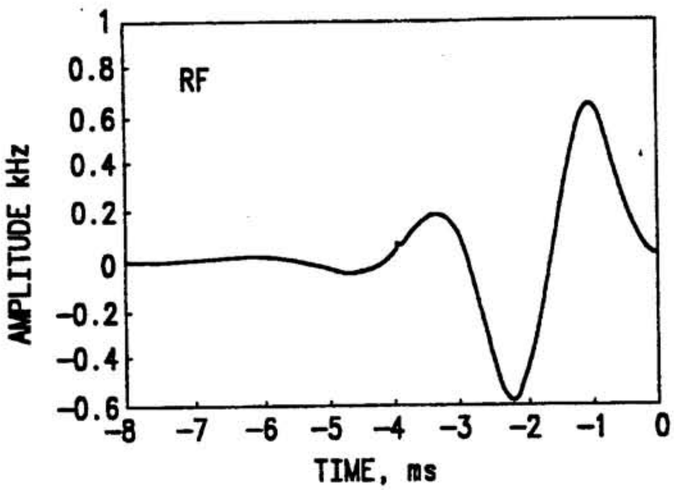
FOR A MINIMUM PHASE B(Z), ONE P(Z) STAGE SUFFICIENT FOR DELAYING ECHO TO END OF THE PULSE

ADDITIONAL DELAY WITH ADDITIONAL STAGES

EACH STAGE ADDS POWER OF A 180!

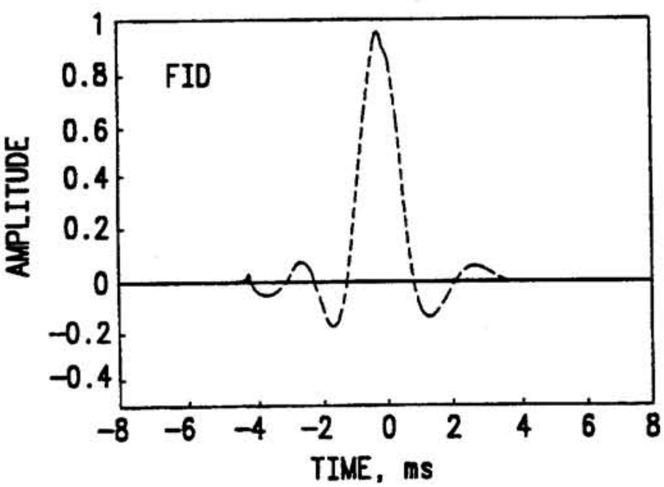
ALSO WORKS FOR B(Z) SCALED TO AN ARBITRARY TIP ANGLE (30, 60, 120...)

True Self Refocusing Pulse



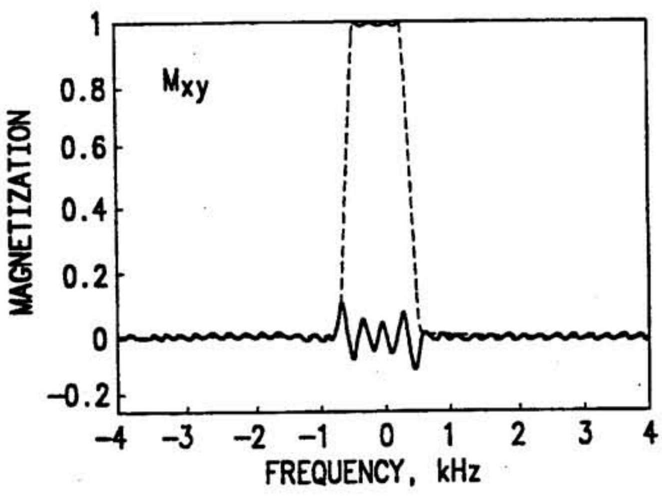
RF Pulse

Based on a TBW=4
minimum phase pulse
2nd Order phase compensation



FID = FT{Slice Profile}

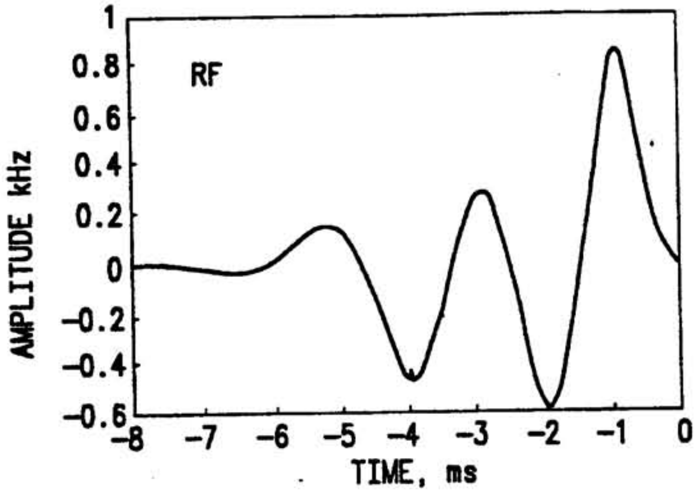
Sinc, centered at t=0



Slice Profile

No crushed transition band

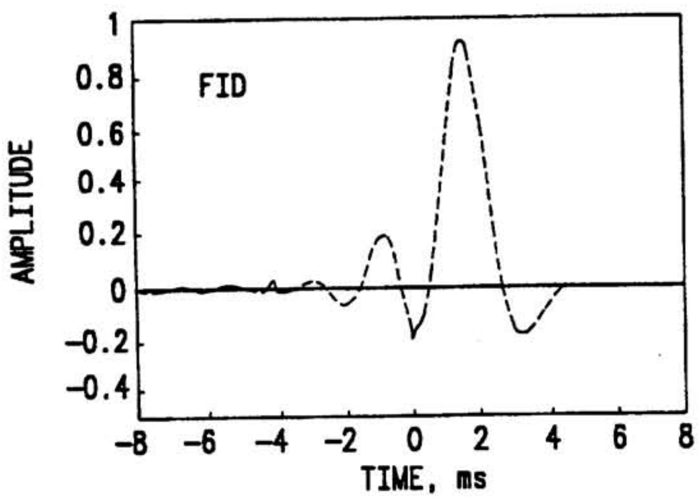
Delayed Refocusing Pulse



RF Pulse

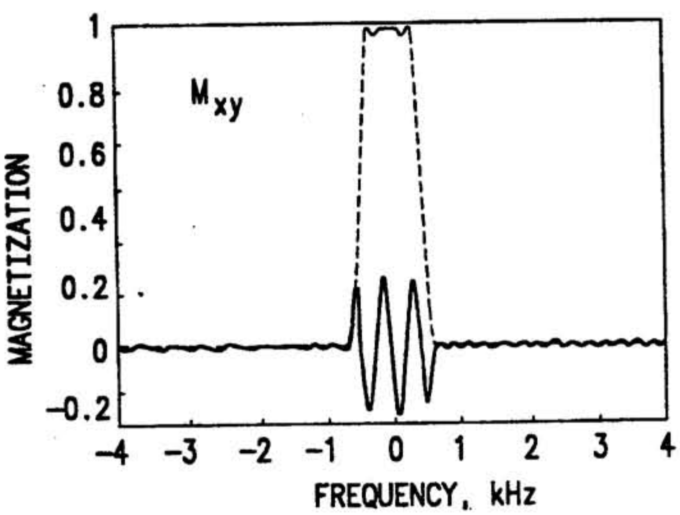
Based on a TBW=4 minimum phase pulse

Two phase compensation stages



FID = FT{Slice Profile}

Sinc, centered at t=1.5 ms

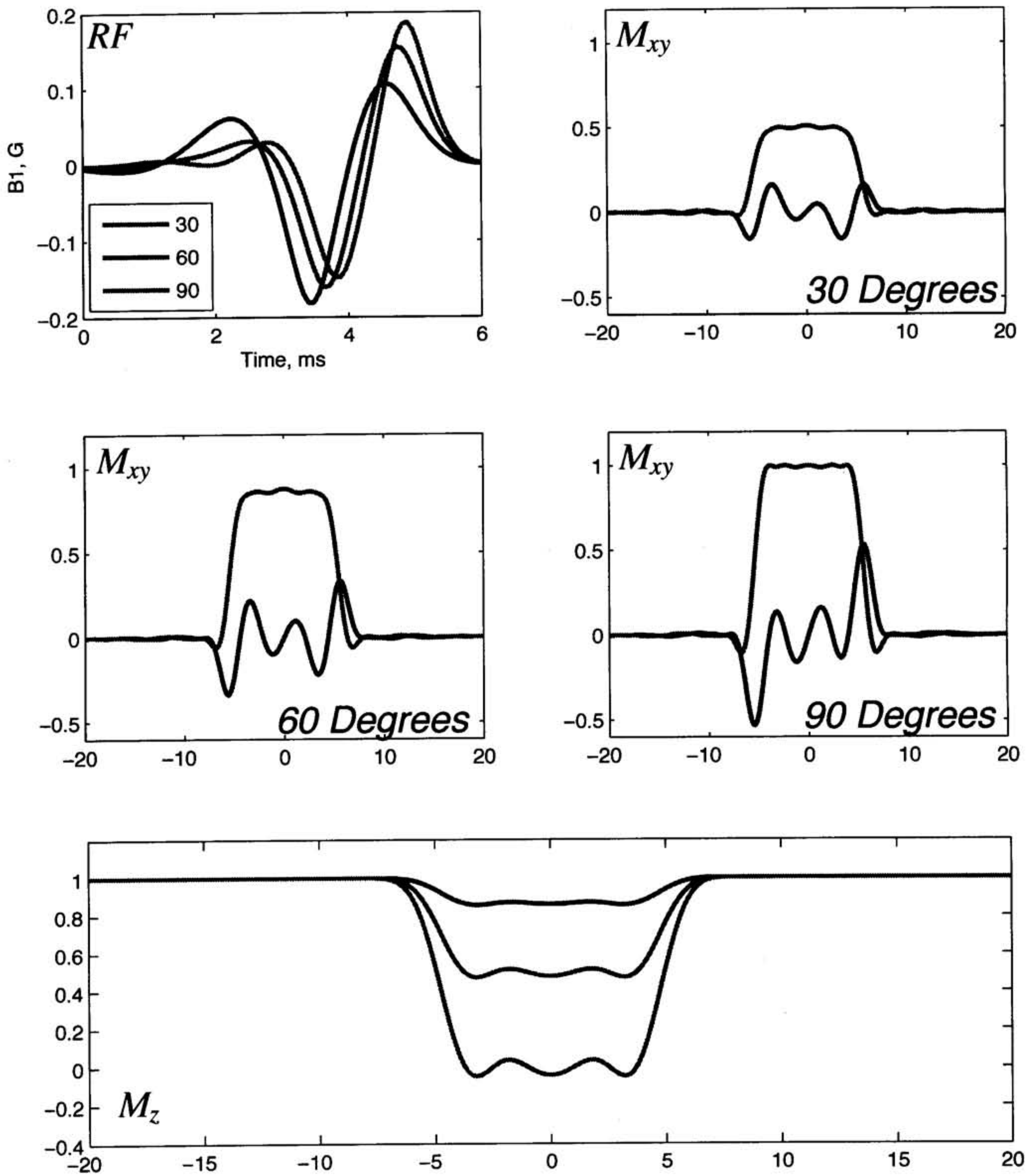


Slice Profile

No crushed transition band

More phase ripple (power constraint)

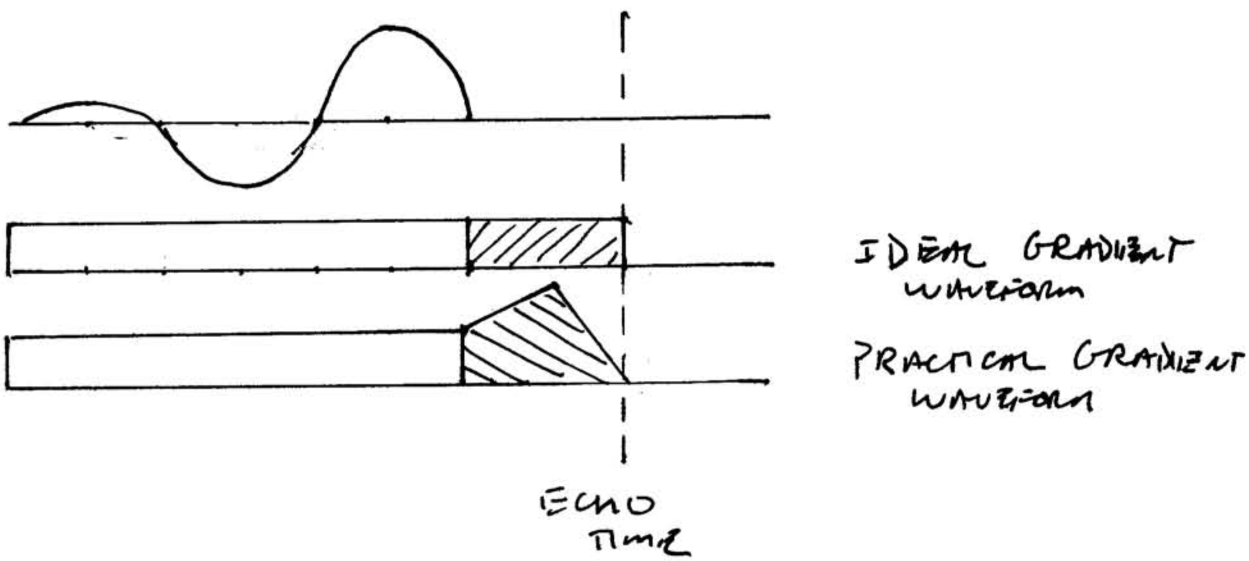
Variable-Tip-Angle SRF Pulses



REFOCUSING REQUIREMENTS

SELF-REFOCUSING PULSES PRODUCE A SPIN-ECHO AT SAME ECHO TIME.

ASSUMES GRADIENT IS ON UNTIL THEN



NEED TO HAVE THE SAME TOTAL GRADIENT AREA AT THE ECHO TIME.

PULSE PAIRS

TRANSVERSE MAGNETIZATION AFTER TWO PULSES

PULSE 1: $\pi/2$

PULSE 2: π

$$m_{xy} = 2\alpha^*\beta$$

$$= \underbrace{\left(2\alpha^*_{\pi/2}\beta_{\pi/2}\right)}_{m_{xy,\pi/2}}\underbrace{\left(\alpha^*_{\pi}\right)^2}_{m_{xy,\pi,CR}} z^2 + \underbrace{\left(\alpha^*_{\pi/2}\alpha_{\pi/2} - \beta_{\pi/2}\beta^*_{\pi/2}\right)}_{m_{z,\pi/2}}\underbrace{\left(2\alpha^*_{\pi}\beta_{\pi}\right)}_{m_{xy,\pi}} z$$

$$\underbrace{- \underbrace{\left(2\alpha_{\pi/2}\beta^*_{\pi/2}\right)}_{m^*_{xy,\pi/2}}\underbrace{\left(\beta_{\pi}\right)^2}_{m_{xy,\pi,SE}}}_{\substack{\text{SPIN ECHO} \\ m_{xy,SE}}}$$

NORMALLY:

$\beta_{\pi/2}$ LINEAR PHASE

β_{π} LINEAR PHASE

$\alpha_{\pi/2}$ ALMOST NO PHASE

$\Rightarrow$ SPIN ECHO IS LINEAR PHASE

WHAT WE REALLY NEED IS

$$m_{xy,SE} = -2\,\alpha_{\pi/2}\,\beta^*_{\pi/2}\,\beta^2_{\pi}$$

IS LINEAR PHASE.

CHOOSE ANY β_{π}

- MINIMUM PHASE
- MINIMUM PEAK RF

CHOOSE

$$\beta_{\pi/2} = \frac{1}{\sqrt{2}}\left(\beta_{\pi}\right)^2$$

THEN

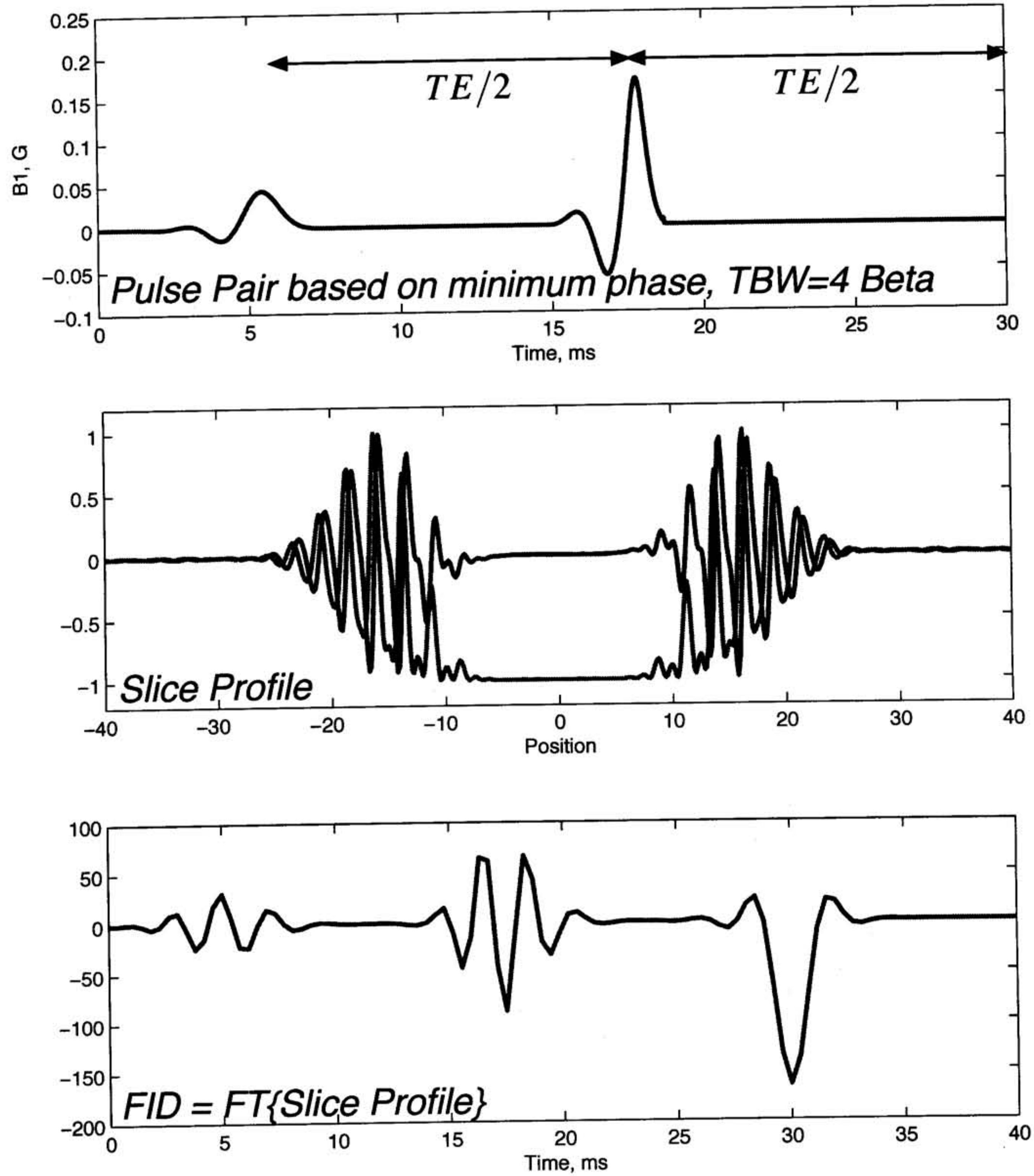
$$m_{xy,SE} = -2\,\alpha_{\pi/2}\left(\frac{1}{\sqrt{2}}\,\beta^2_{\pi}\right)^*\beta_{\pi}$$

$$= -2\,\alpha_{\pi/2}\left(\frac{1}{\sqrt{2}}\right)|\beta_{\pi}|^4$$

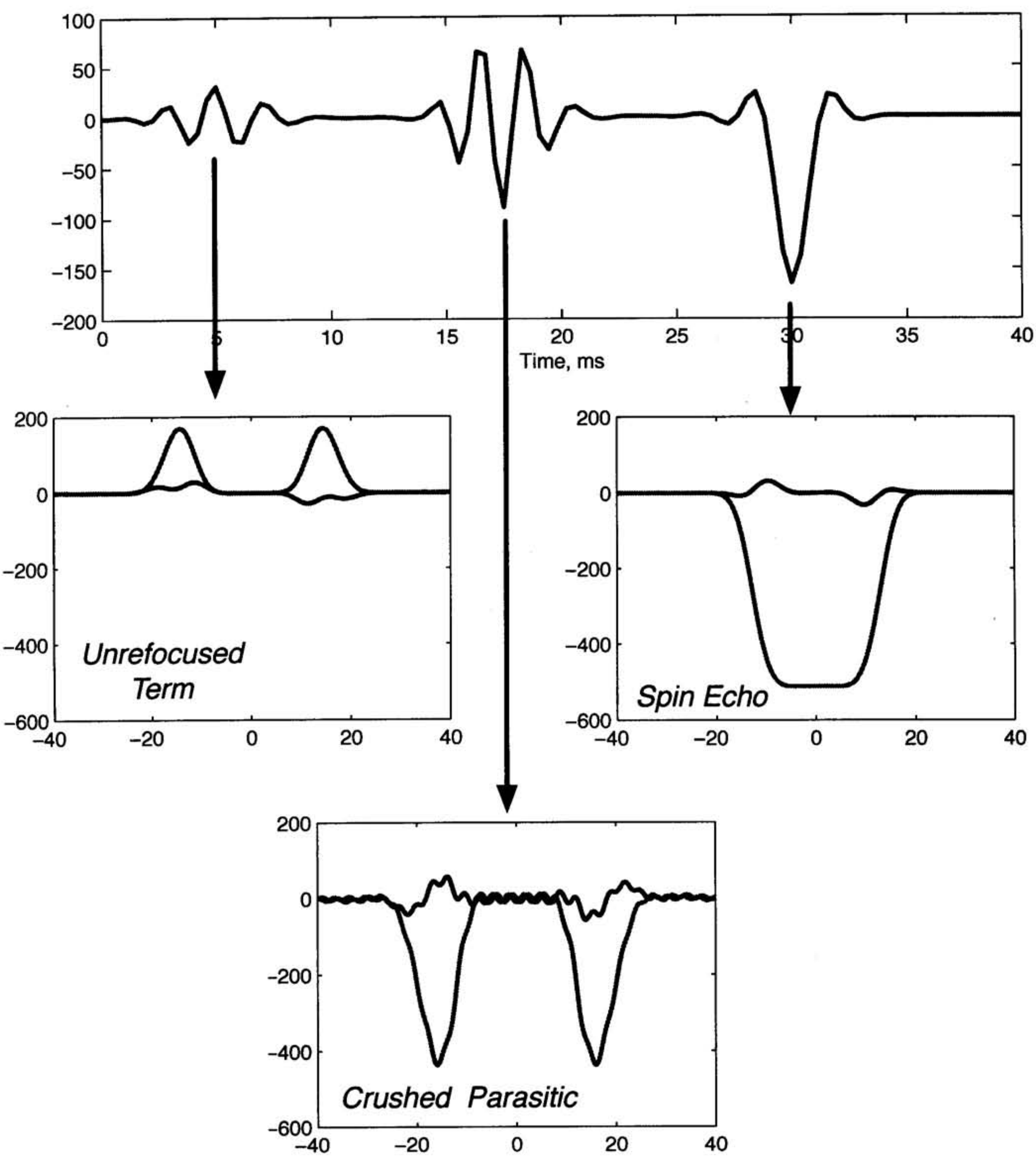
THE $\pi/2$ AND π PULSES PHASE COMPENSATE

LINEAR PHASE ECHO (EXCEPT FOR $\angle\alpha_{\pi/2}$)

Pulse Pair Design



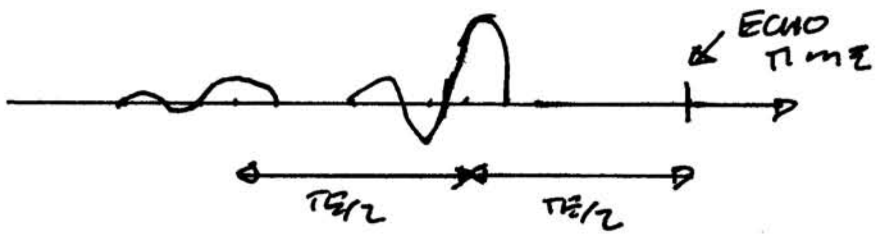
Pulse Pair Terms



PULSE PAIRS BY DIRECT CONSTRUCTION

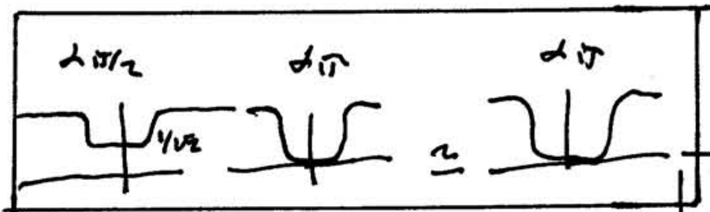
OFTEN WE WANT SHORT ECHO DELAY (1-2 ms)

- TRUE SELF REFOCUSING PULSE TAKE A LOT OF POWER AS ECHO DELAY INCREASES
- PULSE PAIRS HAVE MINIMUM ECHO TIME OF TWO PULSE LENGTHS

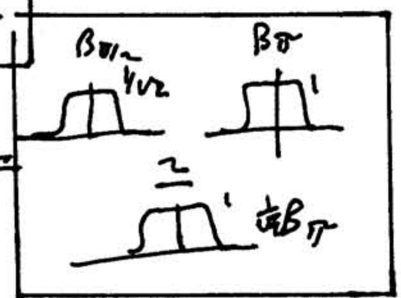


- REALLY WANT SOMETHING IN BETWEEN
    - PULSE PAIR
    - 90 - 180 OVERLAP

α, β FOR TWO PULSE EXPERIMENT

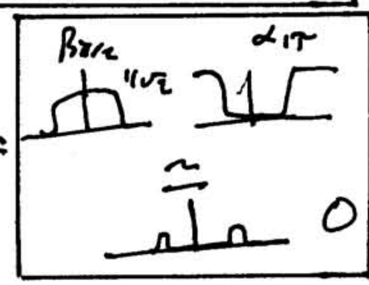


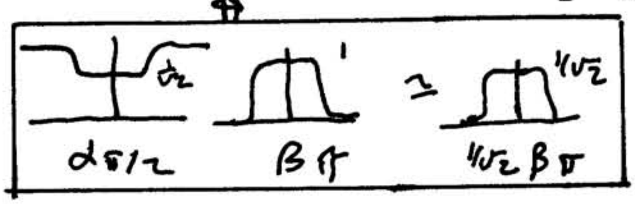
$$\alpha = \alpha_{\pi/2}\alpha_{\pi} z^{-1} - \beta_{\pi/2}\beta_{\pi}^{*}$$



SPIN ECHO TERM

$$\beta = \alpha_{\pi/2}\beta_{\pi} + \beta_{\pi/2}\alpha_{\pi}^{*} z^{+1}$$





SPIN ECHO TERM ($M_{XY} = 2\alpha^{*}\beta$)

$$M_{XY,SE} = -\underbrace{\left(2\alpha_{\pi/2}\beta^{*}_{\pi/2}\right)}_{M^{*}_{XY,\pi/2}}\underbrace{\left(\beta_{\pi}\right)^{2}}_{M_{Y,\pi,SE}}$$

CONSTRUCTION: $\pi/2 - \pi$ PULSE PAIR

1) DESIGN β_π

   MIN PHASE, MIN POWER

2) CONSTRUCT β

$$\beta = \frac{1}{\sqrt{2}} \beta_\pi$$

2) CONSTRUCT α

$$\alpha = \alpha_\pi z^{-1} - \frac{1}{\sqrt{2}} \beta_\pi$$

z^{-1} IS ECHO DELAY

3) DESIGN RF

$$B_1(t) = SLR^{-1}\{\alpha, \beta\}$$

DESIGN OPTIONS

1) ECHO DELAY SET BY z^{-1} TERM

$$z = e^{i\gamma G x(\Delta T)}$$

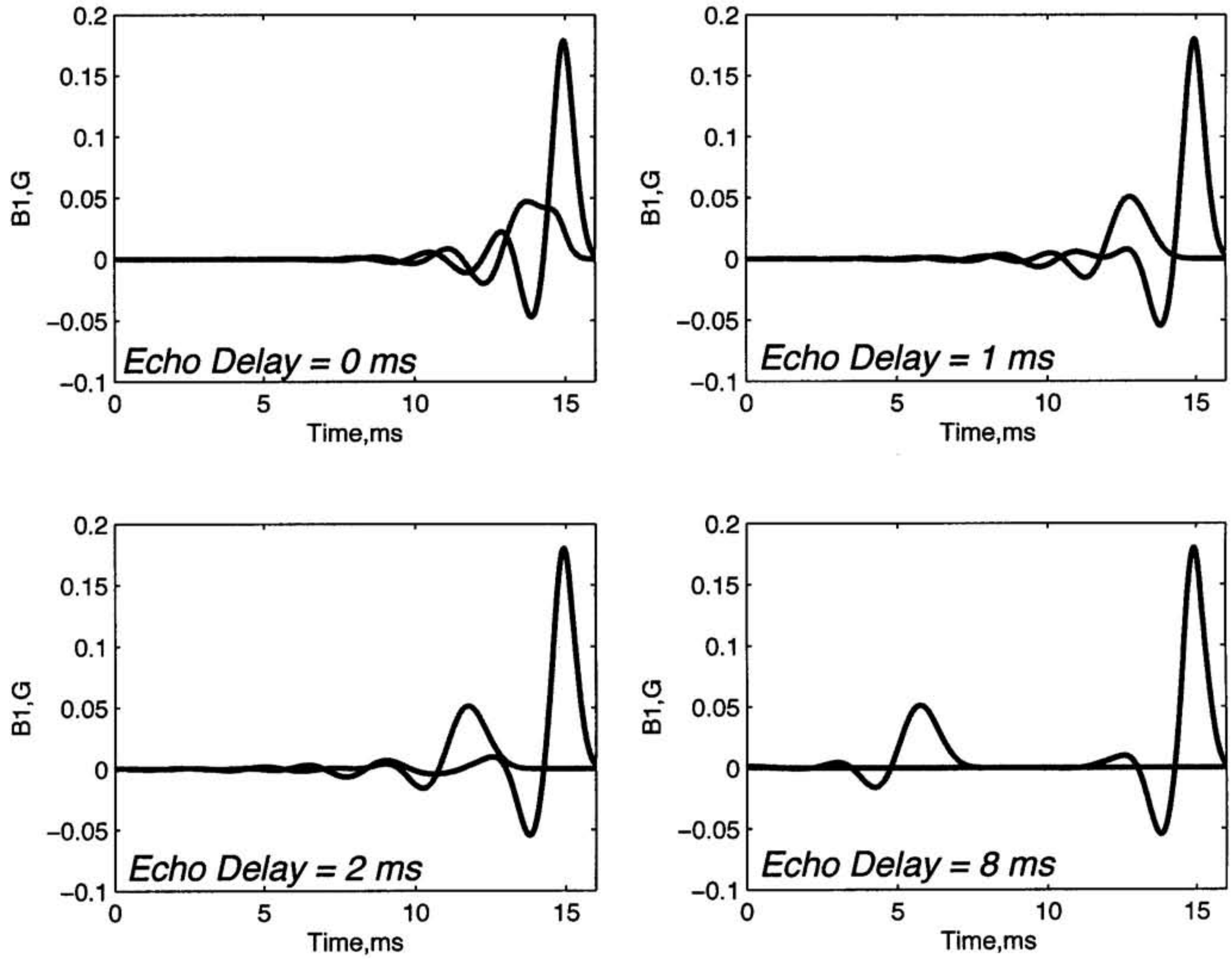
ΔT CAN BE ANYTHING FROM ZERO, UP

2) ANY FLIP ANGLE CAN BE CREATED BY SCALING β TERMS (EXAMPLE IS $\frac{1}{\sqrt{2}} = \sin(\frac{\pi/2}{2})$)

3) ECHO PHASE SET BY ADDING PHASE TO

$$\alpha = \alpha_\pi z^{-1} - \frac{1}{\sqrt{2}} \beta_\pi e^{i\phi}$$

Constructed Pulse Pairs



COMMENTS

1) ZERO ECHO TIME PULSE PAIR IS INDISTINGUISHABLE FROM TRUE SELF REFOCUSING PULSE

2) PARASITIC 180 TERM IS PRESENT, CAN'T BE CRUSHED OUT (NO PLACE FOR CRUSHERS!)

USE PHASE CYCLING OF $\pi/2$.

3) SAME REFOCUSING REQUIREMENTS AS DELAYED REFOCUSING