Lecture 10

LAST TIME

FORWARD AND INVERSE SLR TRANSFORM

TODAY

DESIGNING $B_N(z)$ AND $A_N(z)$

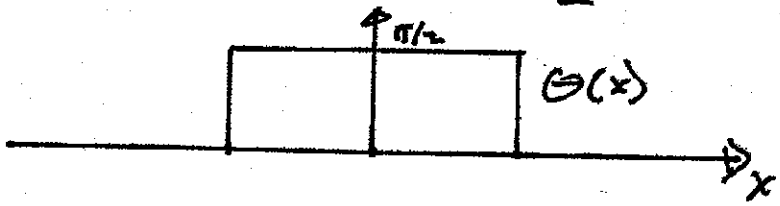
EXAMPLES

TYPES OF $B_N(z)$ DESIGNS

RF PULSE DESIGN WITH SLR

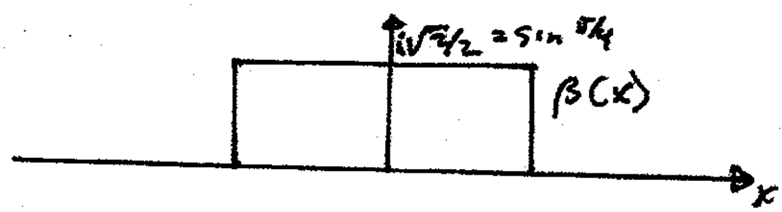
BASIC ALGORITHM ($(\pi/2)_x$ PULSE EXAMPLE)

1) CHOOSE A FLIP ANGLE PROFILE AS A FUNCTION OF SPACE



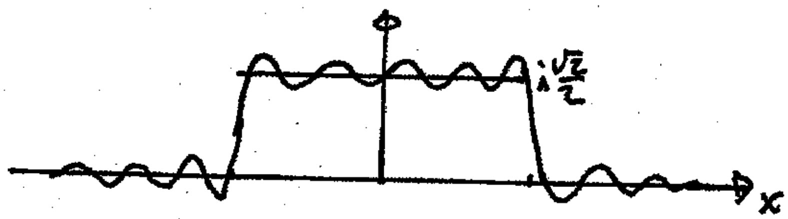
IDEAL FLIP-ANGLE PROFILE
EXCITATION EXAMPLE

2) COMPUTE IDEAL $\beta(x) = \sin \theta(x)/2$



IDEAL β PROFILE

3) APPROXIMATE IDEAL β WITH $B_N(e^{i\gamma G x \Delta t})$



$B_N(z)\big|_{z=e^{i\gamma G x \Delta t}}$

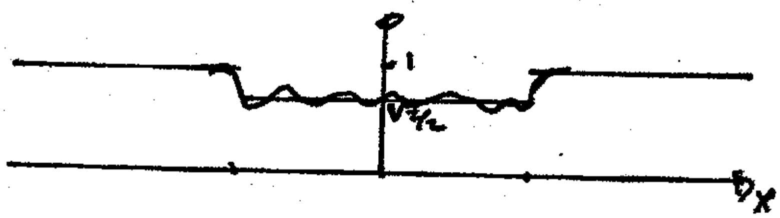
LOWPASS DISCRETE-TIME FILTER

SAMPLED SMALL-TIP-ANGLE FOURIER DESIGN
RF PULSE (WINDOWED SINC)

4) USE THE MAGNITUDE CONSTRAINT

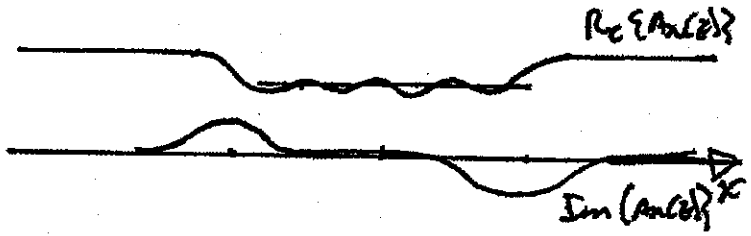
$$|A_N(z)|^2 + |B_N(z)|^2 = 1 \qquad z = e^{i\gamma G x \Delta t}$$

TO SOLVE FOR $|A_N(z)| = \sqrt{1 - |B_N(z)|^2}$



α PROFILE

5) SOLVE FOR THE PHASE OF $A_N(z)$, AND HENCE $A_N(z)$



$A_N(z)$ NOT UNIQUE

MOST USEFUL SOLUTION:

<u>MINIMUM PHASE $A_N(z)$</u>

EASY TO COMPUTE

MINIMUM INTEGRATED POWER

6) USE INVERSE SLR RECURSION TO PRODUCE RF PULSE

MINIMUM PHASE $A_N(z)$

WRITE

$$A_N(z) = |A_N(z)|\, e^{i\angle A_N(z)}$$

COMPLEX LOGARITHM IS

$$\log A_N(z) = \log|A_N(z)| + i\angle A_N(z)$$

IF $A_N(z)$ IS MINIMUM PHASE, NO ZEROS OR POLES ON OR OUTSIDE UNIT CIRCLE, THEN

$$\log A_N(z)$$

IS AN ANALYTIC SIGNAL (ZERO FOR NEGATIVE TIME, THE OTHER DOMAIN)

IN THIS CASE

$$\angle A_N(z) = \mathcal{H}\{\log|A_N(z)|\}$$

WHICH WE CAN COMPUTE DIRECTLY. THEN

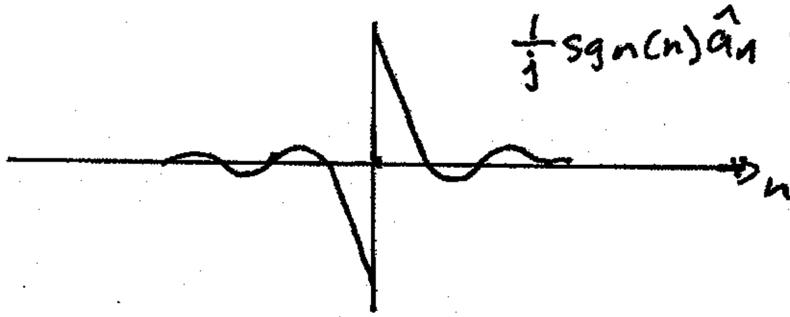
$$A_N(z) = |A_N(z)|\, e^{i\mathcal{H}\{\log|A_N(z)|\}}$$

EASIER APPROACH

GENERATE ANALYTIC SIGNAL DIRECTLY

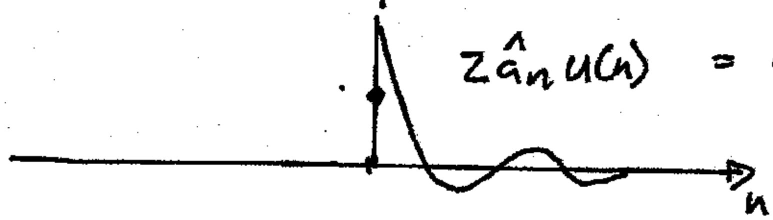
$$\hat{a}_n = DFT^{-1}\{\log|A_N(z)|\}$$

$$\frac{1}{j}\,\mathrm{sgn}(n)\,\hat{a}_n = DFT^{-1}(\mathcal{H}\{\log|A_N(z)|\})$$



$$2\hat{a}_n u(n) = \hat{a}_n + j\left(\tfrac{1}{j}\,\mathrm{sgn}(n)\right)\hat{a}_n$$

$$u(0) = 1/2$$



THEN

$$\log A_N(z) = DFT\{2\hat{a}_n u(n)\}$$

AND

$$A_N(z) = e^{\log A_N(z)} \qquad \text{(COMPLEX)}$$

PRACTICAL ISSUES

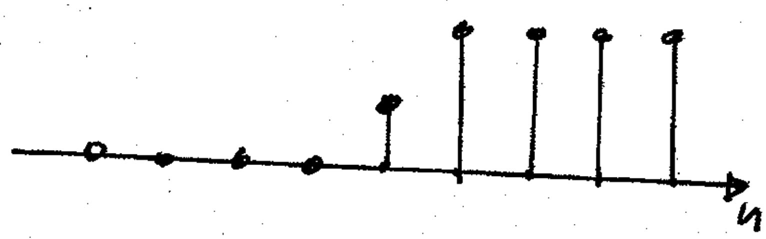
1) EVEN THOUGH $A_N(z)$ IS FINITE ORDER, $\{\hat{a}_n\}$ IS NOT. THE LOGARITHM IS NON-LINEAR. EVALUATE $\{\hat{a}_n\}$ AT SOME HIGHER ORDER.

2) $|A_N(z)|$ MUST BE POSITIVE! NO ZEROS ON UNIT CIRCLE, OR LOG BLOWS UP

$|B_N(z)|$ MUST BE LESS THAN 1

3) THE HILBERT TRANSFORM IS PERFORMED DISCRETELY. THE ORIGIN IS A SPECIAL CASE. TO COMPUTE ANALYTIC SIGNAL

1) DOUBLE POSITIVE TIME SAMPLES
2) KEEP $n=0$ SAMPLE
3) ZERO NEGATIVE SAMPLES



ANOTHER USEFUL PROPERTY OF MINIMUM PHASE SIGNALS:

IF $h_{min}(n)$ IS A MINIMUM PHASE SIGNAL, AND $h(n)$ IS ANY OTHER SIGNAL WITH THE SAME MAGNITUDE SPECTRUM $|H(z)|$ AND ANOTHER PHASE SPECTRUM $\angle H(z)$, THE ENERGY IN THE MINIMUM PHASE SIGNAL IS MORE CONCENTRATED NEAR THE ORIGIN

$$\sum_{n=0}^{m} |h(n)|^2 \leq \sum_{n=0}^{m} |h_{min}(n)|^2 \quad \text{FOR ALL } m$$

IN PARTICULAR

$$|h(0)|^2 \leq |h_{min}(0)|^2$$

$$|h(0)| \leq |h_{min}(0)|$$

THE MINIMUM PHASE SIGNAL HAS THE LARGEST VALUE AT $n=0$.

MINIMUM RF POWER

MINIMUM PHASE $A_N(z)$ HAS THE LARGEST CONSTANT COEFFICIENT $A_{N,0}$.

THE FORWARD RECURSION IS

$$\begin{pmatrix} A_j(z) \\ B_j(z) \end{pmatrix} = \begin{pmatrix} C_j & -S_j^* z^{-1} \\ S_j & C_j z^{-1} \end{pmatrix} \begin{pmatrix} A_{j-1}(z) \\ B_{j-1}(z) \end{pmatrix}$$

THE CONSTANT COEFFICIENT IS THEN

$$A_{N,0} = C_N C_{N-1} \cdots C_2 C_1$$

FOR SMALL INCREMENTAL TIP ANGLES

$$C_j = \cos \theta_j/2 \simeq 1 - \tfrac{1}{2}\left(\tfrac{\theta_j}{2}\right)^2$$

$$= 1 - \tfrac{1}{8}\theta_j^2$$

THEN

$$A_{N,0} = \left(1 - \tfrac{1}{8}\theta_n^2\right)\left(1 - \tfrac{1}{8}\theta_{n-1}^2\right) \cdots \left(1 - \tfrac{1}{8}\theta_2^2\right)\left(1 - \tfrac{1}{8}\theta_1^2\right)$$

$$= 1 - \tfrac{1}{8}\sum_{j=0}^{N} \theta_n^2 + \underbrace{\cdots}_{\text{HIGHER ORDER TERMS} \sim \frac{1}{N^2} \text{ AND FASTER}}$$

$\sim \frac{1}{N}$

HIGHER ORDER TERMS $\sim \frac{1}{N^2}$ AND FASTER

RECALL

$$\theta_j = \gamma |B_{1,j}| \Delta t$$

SO

$$A_{N,0} = 1 - \frac{1}{8} (\gamma \Delta t)^2 \underbrace{\sum_{j=0}^{N} |B_{1,j}|^2}_{\text{RF POWER}}$$

LARGEST $A_{N,0}$ MEANS SMALLEST RF POWER

MINIMUM PHASE $A_N(z)$ GIVES MINIMUM POWER $B_1(t)$

<u>ALMOST ALWAYS WHAT YOU WANT</u>

PULSE DESIGN IS DETERMINED BY $B_N(z)$

$A_N(z)$ IS CHOSEN TO BE CONSISTANT, MINIMUM POWER

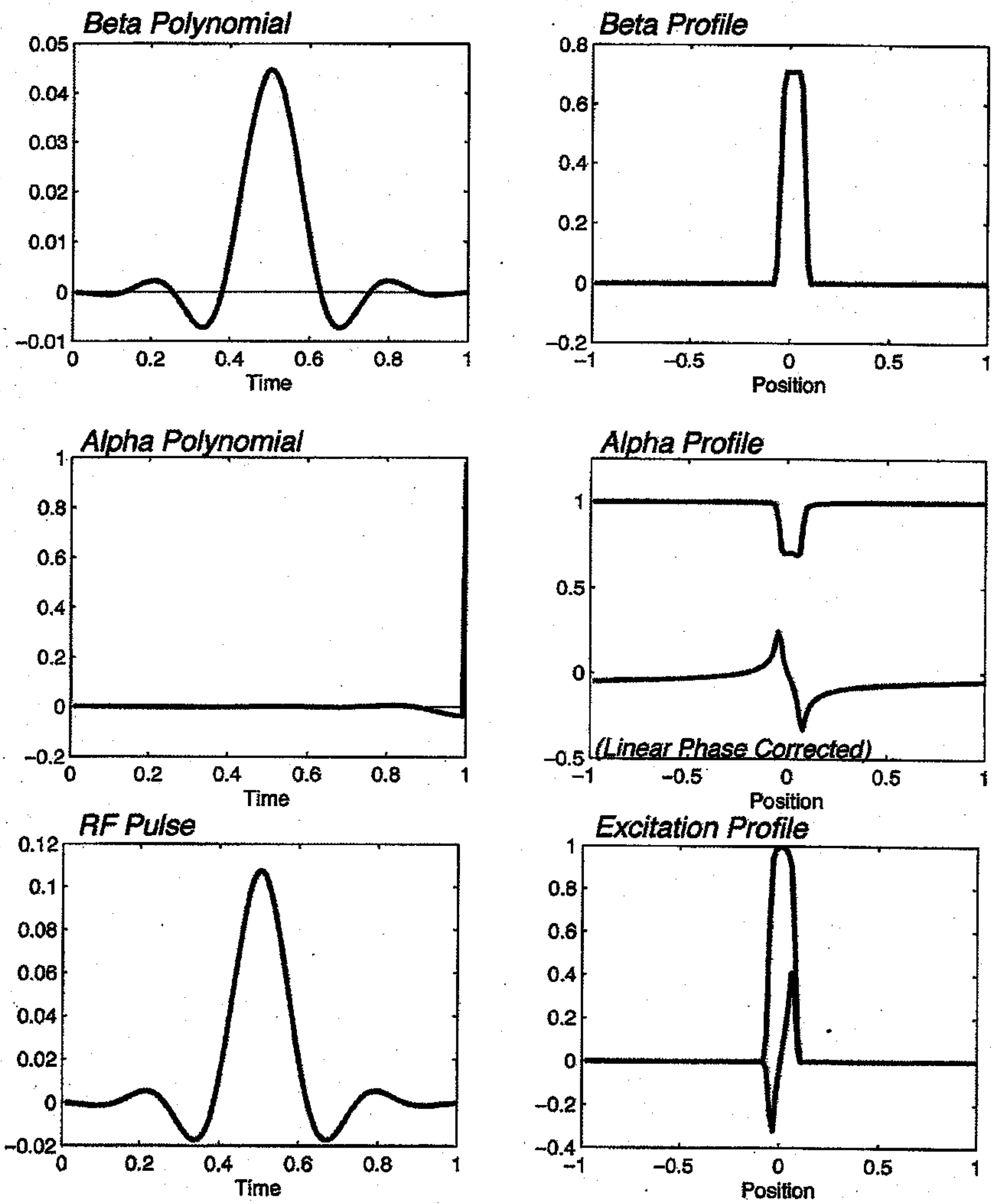
<u>EXCEPTION</u> SELF REFOCUSING PULSES

PHASE ADDED TO α SO THAT

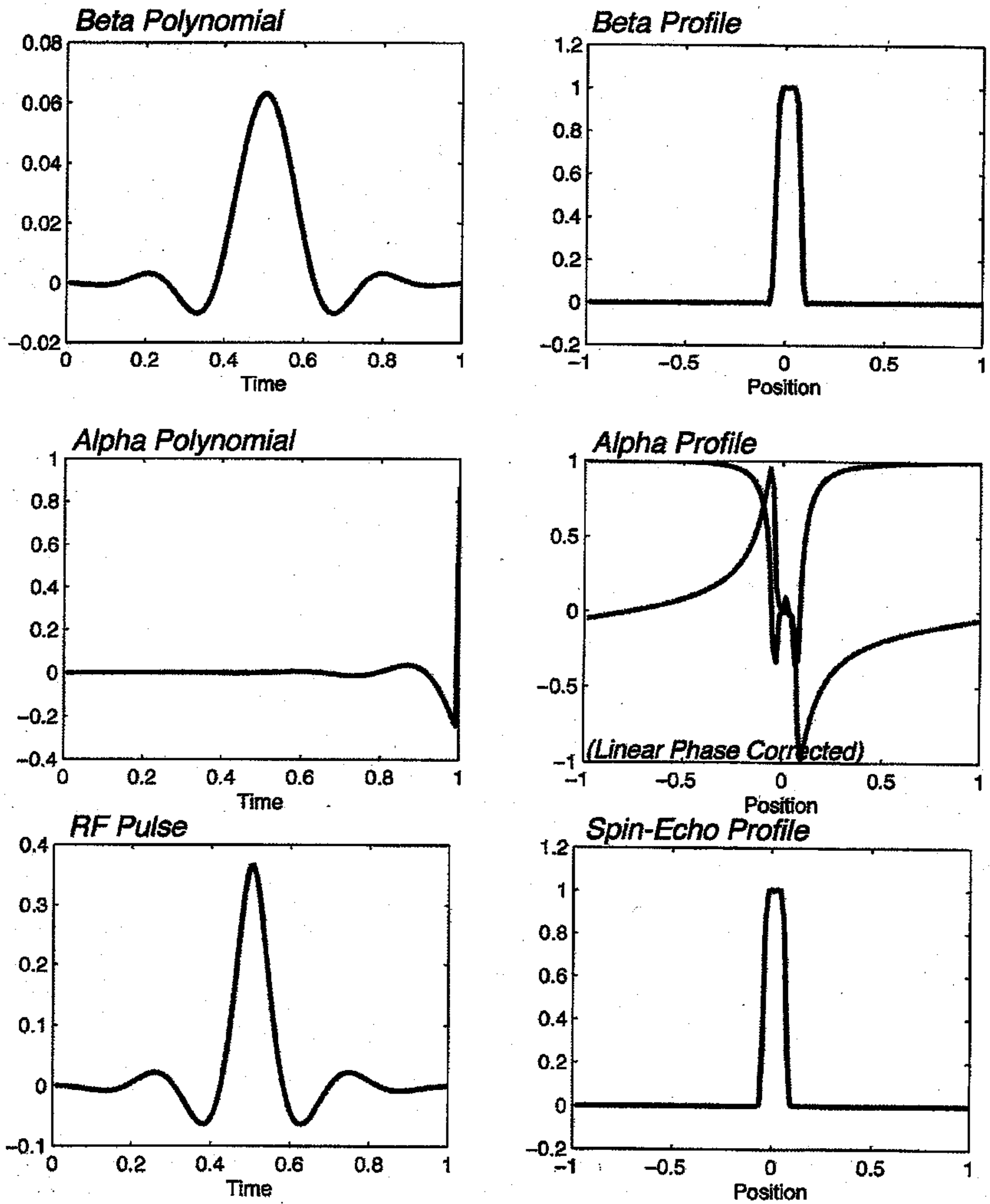
$$2\alpha^* \beta$$

HAS ENOUGH PHASE TO SHIFT THE ECHO TO THE END OF THE PULSE OR BEYOND. VERY EXPENSIVE IN RF POWER!

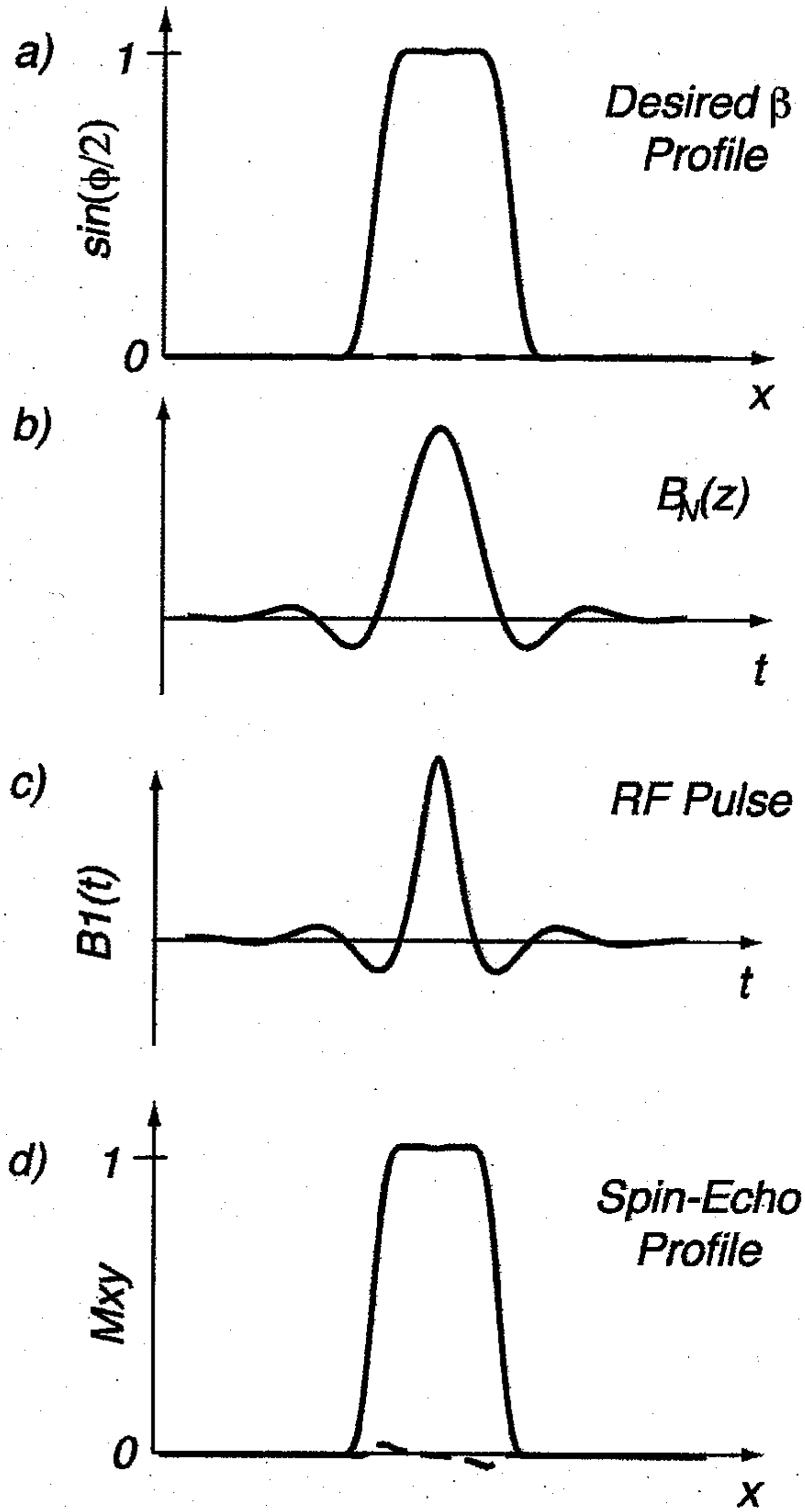
SLR Excitation Pulse Design



SLR Spin-Echo Pulse Design



SLR Spin-Echo Pulse Design



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TYPES OF $B_N(z)$ DESIGNS

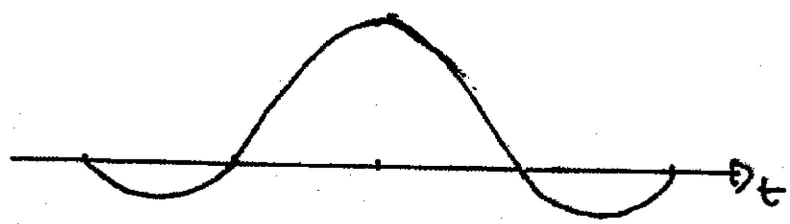
MANY DIFFERENT OPTIONS FOR $B_N(z)$

LINEAR PHASE: MOST COMMON

PERFECTLY REFOCUSED WITH GRADIENT REVERSAL AS AN EXCITATION PULSE

SPIN ECHO PULSES

SYMMETRIC IN TIME

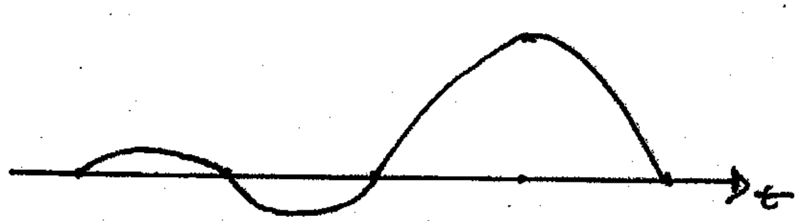


ALSO MAXIMUM PEAK POWER (PERFECTLY REPHASES!)

NOT THE MOST SELECTIVE

MINIMUM PHASE SAT PULSES AND INVERSIONS

THE FLIP OCCURS AS LATE IN THE PULSE AS POSSIBLE



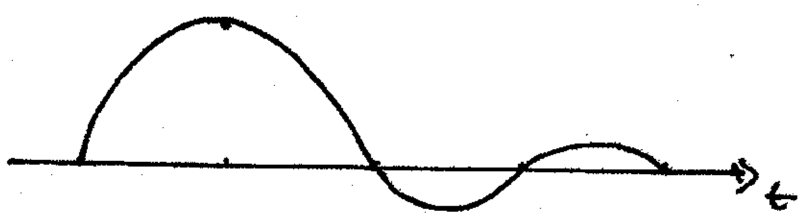
MOST SELECTIVE PULSES

DOES NOT PERFECTLY REFOCUS

ALMOST THE SAME PEAK POWER AS LINEAR PHASE

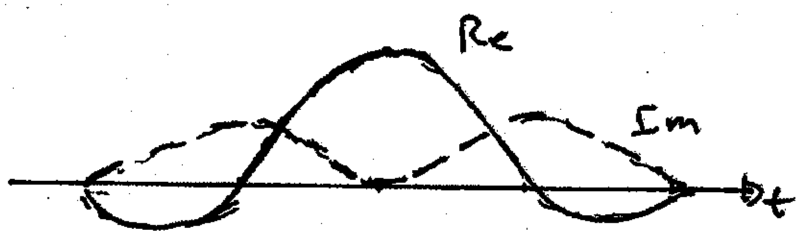
<u>MAXIMUM PHASE</u> SATURATION AND INVERSION

MINIMUM PHASE PULSE REVERSED



<u>QUADRATIC OR NONLINEAR PHASE</u>

SPREADS RF POWER OUT



IDENTICAL TOTAL POWER AS MIN/MAX PHASE PULSE WITH SAME PROFILE

<u>MUCH</u> LOWER PEAK POWER