Lecture 12

ASSIGNMENT

READ "ADIABATIC EXCITATION AND INVERSION PULSES"
SECTIONS 6.1, 6.2 PAGES 177-199

TODAY

ADIABATIC INVERSION PULSES

NEXT TIME

ADIABATIC ROTATIONS

ADIABATIC PULSES

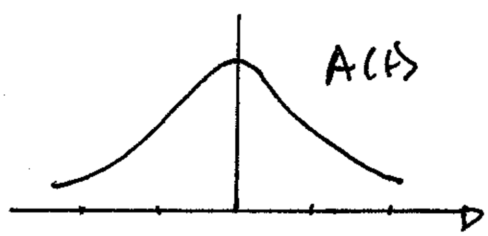
FREQUENCY MODULATED PULSES OF THE FORM

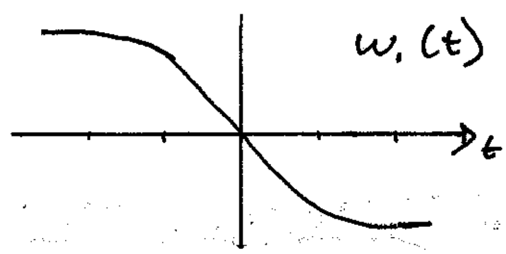
$$B_1(t) = A(t)\, e^{-i\omega_1(t)t}$$

WHERE

$A(t)$ - ENVELOPE

$\omega_1(t)$ - FREQUENCY SWEEP





MANY DIFFERENT OPTIONS. MOST FAMOUS

$$A(t) = A_0 \operatorname{sech}(\beta t)$$

$$\omega_1(t) = -\mu\beta \tanh(\beta t)$$

μ, β PARAMETERS. <u>SILVER-HOULT</u> PULSE, OR <u>HYPERBOLIC SECANT</u>.

ONE OF THE FEW ANALYTIC SOLUTIONS TO THE BLOCH EQUATION

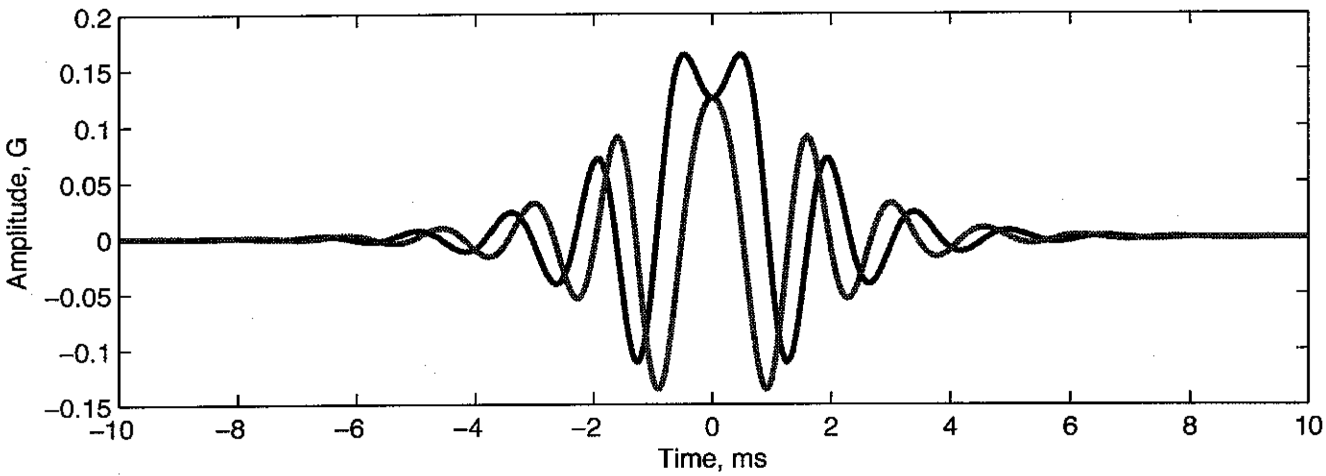
FIRST OF A FAMILY OF SOLUTIONS CALLED SOLITONS.

REMARKABLE CHARACTERISTIC

A HYPERBOLIC SECANT PULSE PERFORMS A FREQUENCY (SLICE) SELECTIVE INVERSION FOR ANY RF AMPLITUDE ABOVE A THRESHOLD.

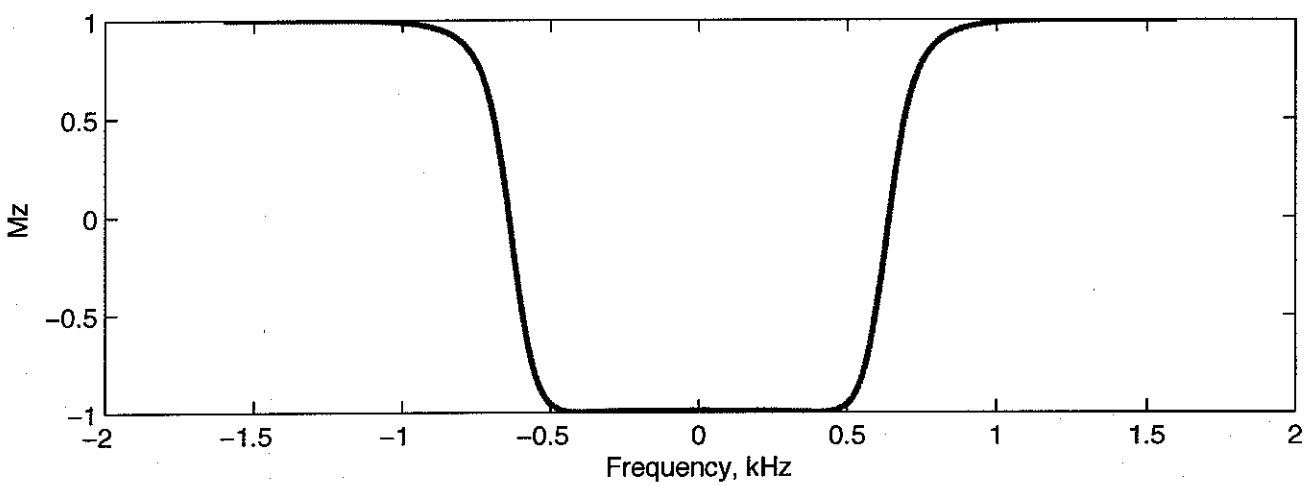
Hyperbolic Secant

RF Pulse

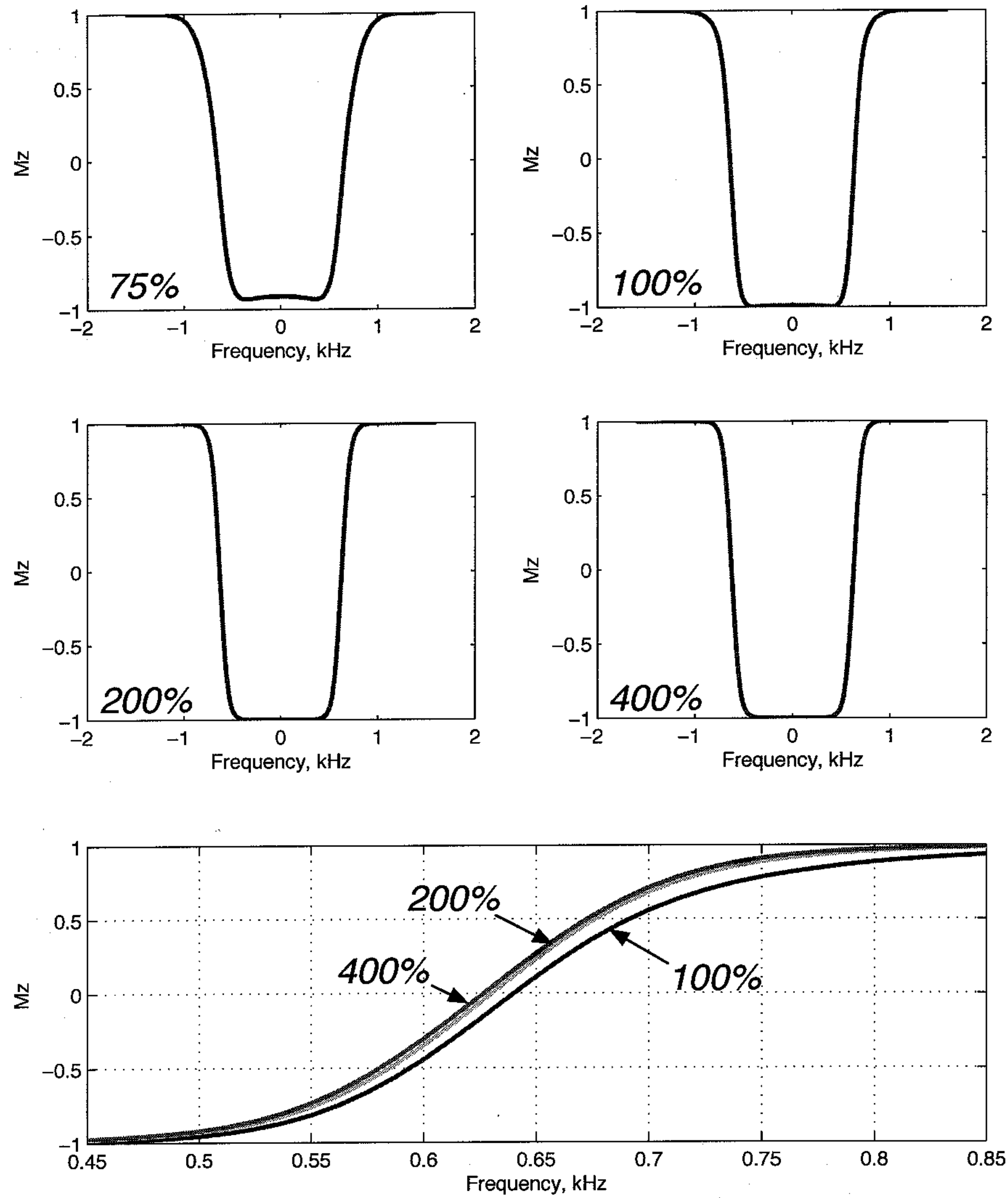


$$B_1(t) = A_0 sech(\beta t) e^{-i\mu\beta \tanh(\beta t) t}$$
$$\beta = 800\ rad/s; \quad \mu = 4.9$$

Inversion Profile



Inversion Profiles



ROTATING REFERENCE FRAMES

BLOCH EQUATION IN LABORATORY FRAME

$$\left(\frac{d\underline{m}}{dt}\right) = -\gamma \underline{B} \times \underline{m}$$

WHERE

$$\underline{B} = B_1 \cos\omega_0 t\, \hat{\imath}_0 + B_1 \sin\omega_0 t\, \hat{y}_0 + B_0 \hat{k}_0$$

AND $\hat{\imath}_0, \hat{\jmath}_0, \hat{k}_0$ ARE FIXED UNIT VECTORS IN LAB FRAME, AND $B_1 = B_{1,x}$ FOR CONVENIENCE

WE WANT TO DESCRIBE THE MAGNETIZATION IN A FRAME ROTATING AT ω_0

$$\hat{\imath} = \hat{\imath}_0 \cos\omega_0 t + \hat{\jmath}_0 \sin\omega_0 t$$

$$\hat{\jmath} = -\hat{\imath}_0 \sin\omega_0 t + \hat{\jmath}_0 \cos\omega_0 t$$

$$\hat{k} = \hat{k}_0$$

THEN

$$\underline{B} = B_1 \hat{\imath} + B_0 \hat{k}$$

$$\underline{m} = m_x \hat{\imath} + m_y \hat{\jmath} + m_z \hat{k}$$

BOTH (m_x, m_y, m_z) AND $\hat{\imath}, \hat{\jmath}, \hat{k}$ TIME VARYING IN LAB FRAME.

DIFFERENTIATING

$$\left(\frac{d}{dt}\underline{m}\right)_{LAB} = \frac{\partial m_x}{\partial t}\hat{\imath} + \frac{\partial m_y}{\partial t}\hat{\jmath} + \frac{\partial m_z}{\partial t}\hat{k}$$

$$+ m_x\frac{\partial \hat{\imath}}{\partial t} + m_y\frac{\partial \hat{\jmath}}{\partial t} + m_z\frac{\partial \hat{k}}{\partial t}$$

$$= \left(\frac{\partial \underline{m}}{\partial t}\right)_{ROT} + m_x\frac{\partial \hat{\imath}}{\partial t} + m_y\frac{\partial \hat{\jmath}}{\partial t} + m_z\frac{\partial \hat{k}}{\partial t}$$

$\hat{\imath}, \hat{\jmath}$ ARE ROTATING UNIT VECTORS WHICH WE CAN REPRESENT AS A CROSS PRODUCT WITH

$$\underline{\omega}_0 = \omega_0\hat{\underline{k}}$$

SO

$$\frac{\partial \hat{\imath}}{\partial t} = \underline{\omega}_0 \times \hat{\imath} \; ; \quad \frac{\partial \hat{\jmath}}{\partial t} = \underline{\omega}_0 \times \hat{\jmath} \; ; \quad \frac{\partial \hat{k}}{\partial t} = \underline{\omega}_0 \times \hat{k}$$

SUBSTITUTING

$$\left(\frac{d}{dt}\underline{m}\right)_{LAB} = \left(\frac{\partial \underline{m}}{\partial t}\right)_{ROT} + \underline{\omega}_0 \times \left(m_x\hat{\imath} + m_y\hat{\jmath} + m_z\hat{k}\right)$$

$$= \left(\frac{\partial m}{\partial t}\right)_{ROT} + \underline{\omega}_0 \times \underline{m}$$

IN THE ROTATING FRAME

$$\left(\frac{\partial \underline{m}}{\partial t}\right)_{ROT} = \frac{d}{dt}\underline{m} - \underline{\omega_0} \times \underline{m}$$

$$= -\gamma \underline{B} \times \underline{m} - \underline{\omega_0} \times \underline{m}$$

$$= \left(-\gamma \underline{B} - \underline{\omega_0}\right) \times \underline{m}$$

$$= -\gamma \underbrace{\left(\underline{B} + \frac{\underline{\omega_0}}{\gamma}\right)}_{\underline{B}_{eff}} \times \underline{m}$$

AT THE LARMOR FREQUENCY

$$\underline{\omega_0} = -\gamma B_0 \hat{k}$$

AND

$$\underline{B} = B_1 \cos\omega_0 t\, \hat{i}_0 + B_1 \sin\omega_0 t\, \hat{j}_0 + B_0 \hat{k}_0$$

$$= B_1 \hat{i} + B_0 \hat{k}$$

SO

$$\underline{B}_{eff} = B_1 \hat{i} + \cancel{B_0 \hat{k}} + \cancel{\frac{-\gamma B_0}{\gamma} \hat{k}}$$

$$= B_1 \hat{i}$$

THE RESULT IS THAT THE STATIC B_0 FIELD HAS BEEN "DELETED" IN ROTATING FRAME!

KEY IDEA

CHANGING REFERENCE FRAMES BY A FREQUENCY ω CORRESPONDS TO ADDING A Z-FIELD OF $\frac{\omega}{\gamma}$.

IN PARTICULAR IF I HAVE AN RF PULSE

$$B_1(t) = A(t)\, e^{i\omega_1(t)\, t}$$

AT THE LARMOR FREQUENCY, THEN

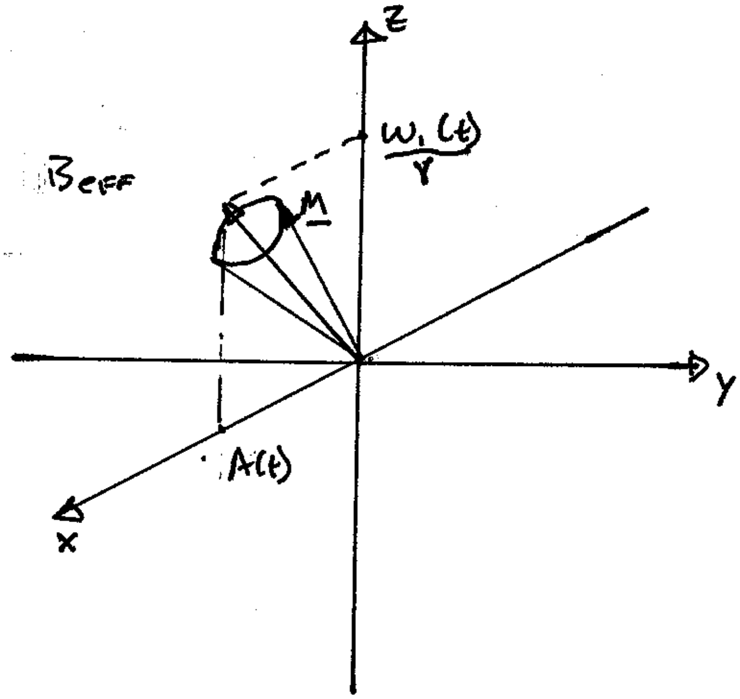
$$(\underline{B})_{\omega_0} = A(t)\cos\omega_1(t)\,\hat{i}_{\omega_0} + A(t)\sin\omega_1(t)\,\hat{j}_{\omega_0} + 0\,\hat{k}$$

IF WE CHANGE TO THE ROTATING FRAME AT $\omega_0+\omega_1(t)$

$$(\underline{B})_{\omega_0+\omega_1(t)} = A(t)\,\hat{i}_{\omega_0+\omega_1(t)} + \frac{\omega_1(t)}{\gamma}\hat{k}$$

$$= \left(A(t),\ 0,\ \frac{\omega_1(t)}{\gamma}\right)$$

MAGNETIZATION PLOT



MAGNETIZATION PRECESSES ABOUT $\underline{B}_{eff}$

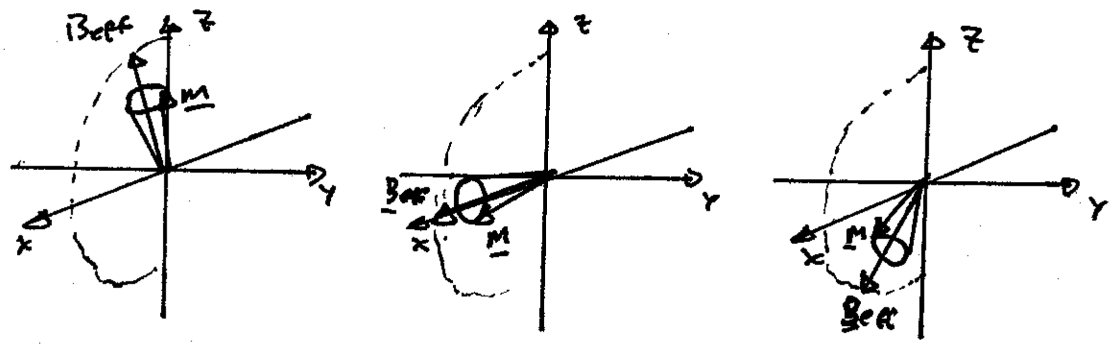
$$\underline{B}_{eff} = \left(A(t),\ 0,\ \frac{\omega_1(t)}{\gamma} \right)$$

$$|B_{eff}| = \sqrt{A^2(t) + \left(\frac{\omega_1(t)}{\gamma}\right)^2}$$

$$\angle B_{eff} = \psi = \tan^{-1}\left(\frac{A(t)}{\omega_1(t)/\gamma}\right)$$

BASIC IDEA OF ADIABATIC PULSES

GIVEN INITIAL $\underline{M}$ ALONG +Z AXIS, SWEEP $\underline{B}_{eff}$ FROM +Z TO −Z SLOWLY ENOUGH THAT $\underline{M}$ <u>FOLLOWS</u> $\underline{B}_{eff}$



$\underline{M}$ SHOULD PRECESS <u>FASTER</u> THAN THE ANGLE OF $\underline{B}_{eff}$ CHANGES, $\frac{d\varphi}{dt}$

$$\left|\frac{d\varphi}{dt}\right| \ll \gamma \left|B_{eff}\right|$$

<u>ADIABATIC</u> <u>CONDITION</u>. DEFINE

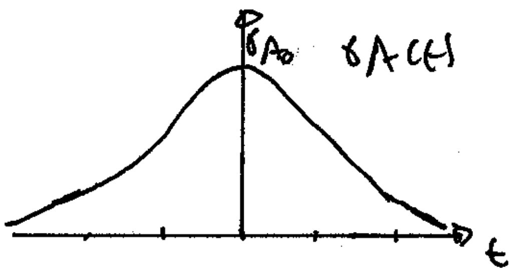
$$\eta = \frac{\gamma \left|B_{eff}\right|}{\left|\frac{d\varphi}{dt}\right|}$$

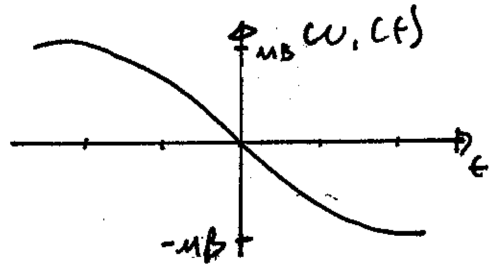
AS <u>ADIABATIC</u> <u>FACTOR</u>, LARGER THAN 1.

Sweep Diagrams

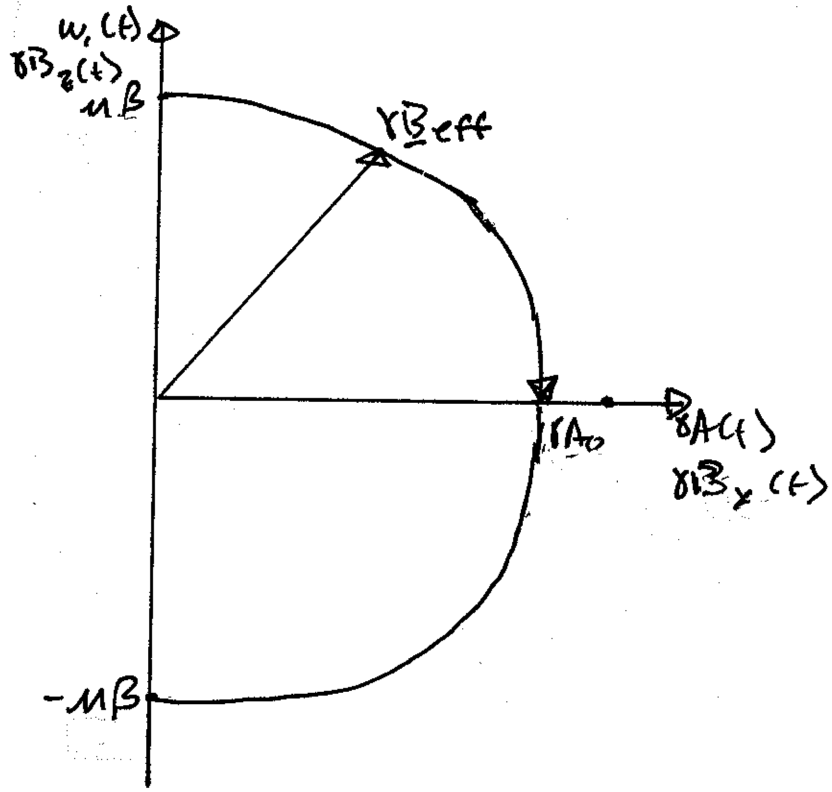
Very usefull tool for understanding adiabatic pulses. Due to Conolly.

Plot $\gamma A(t)$ vs $\omega_1(t)$

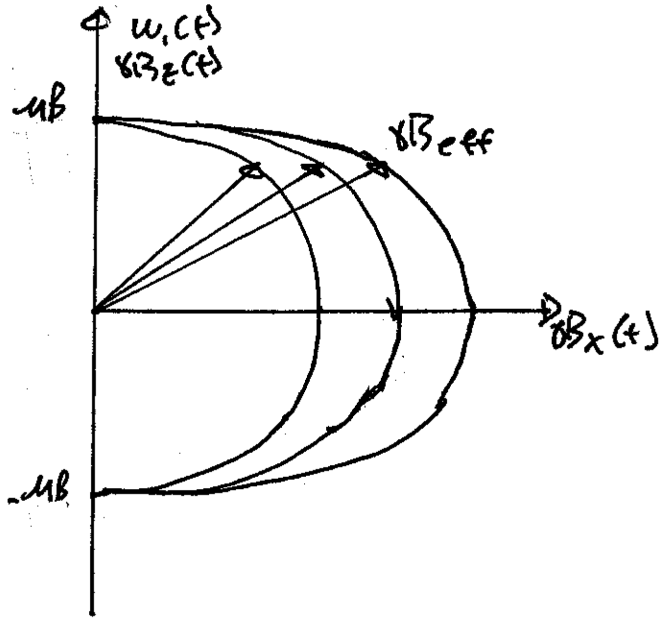




$\gamma A(t) = \gamma A_0 \, \mathrm{sech}(\beta t)$ $\qquad$ $\gamma \omega_1(t) = -\mu \beta \tanh(\beta t)$



ADIABATIC PULSE ARE INSENSITIVE TO B_1 SCALING, PROVIDED ADIABATIC CONDITION IS MET



ALL PRODUCE INVERSIONS PROVIDED

$$\frac{d\varphi}{dt} \ll \gamma |B_{eff}|$$

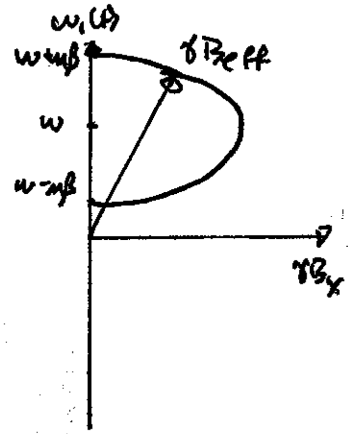
AS B_1 INCREASES, CAN LOOSE ADIABATICITY AT THE BEGINNING AND ENDS.

ADIABATICITY IMPROVES IN THE MIDDLE.

ADIABATIC PULSES ARE SELECTIVE

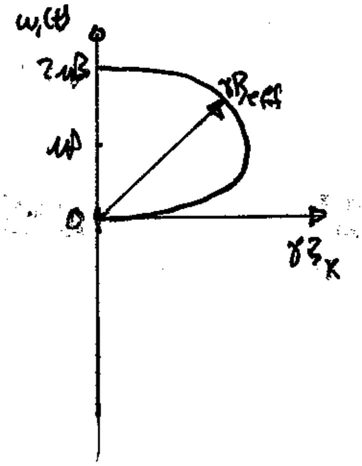
FREQUENCY OR SLICE SELECTIVE

DETERMINED BY SWEEP BANDWIDTH



$\omega > \mu\beta$

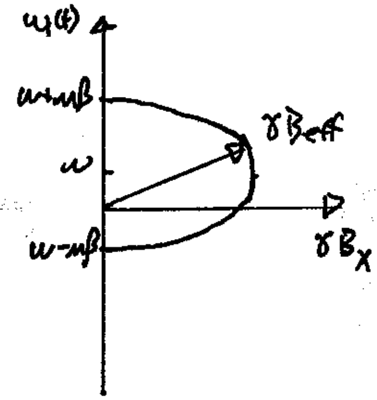
NOT INVERTED



$\omega = \mu\beta$

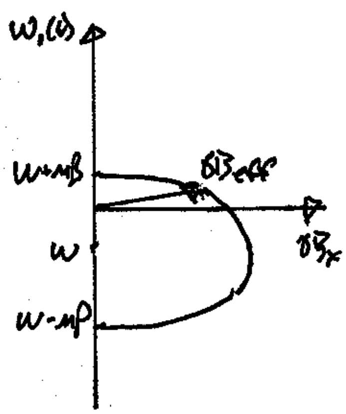
TRANSITION BAND

(NOT ADIABATIC)



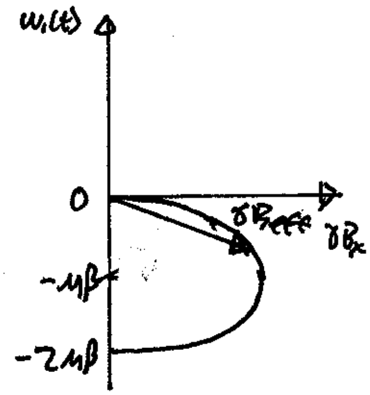
$-\mu\beta < \omega < \mu\beta$

INVERTED



$-\mu\beta < \omega < \mu\beta$

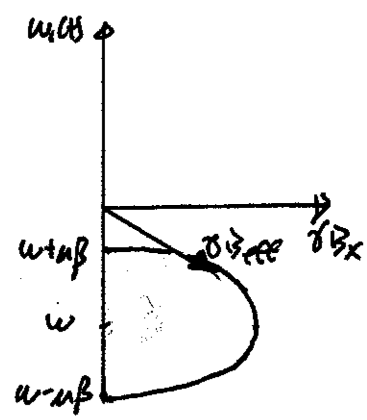
INVERTED



$\omega = -\mu\beta$

TRANSITION BAND

(NOT ADIABATIC)



$\omega < -\mu\beta$

NOT INVERTED

ANTI-PARALLEL

FOR $\omega > \mu\beta$ OR $\omega < -\mu\beta$, $\underline{M}$ RETURNED TO $+z$ AXIS

FOR $-\mu\beta < \omega < \mu\beta$ $\underline{M}$ IS INVERTED

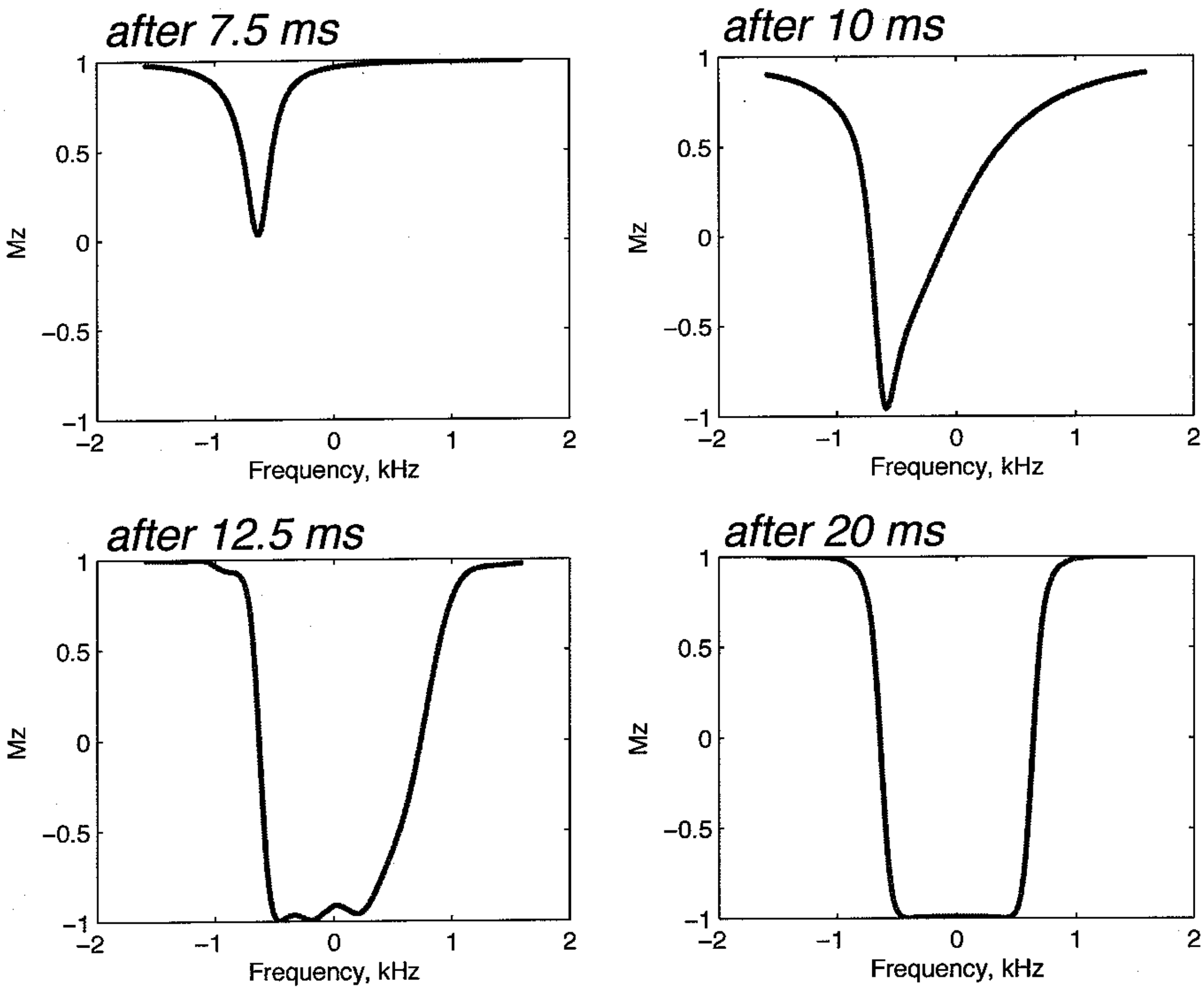
BANDWIDTH IS

$$\Delta f = \frac{2\mu\beta}{2\pi}$$

$$= \underline{\frac{\mu\beta}{\pi}} \quad \text{Hz}$$

BAND EDGE CLOSEST TO BEGINNING OF FREQUENCY SWEEP DEFINED FIRST, OTHER BAND EDGE LAST

Inversion Sequence



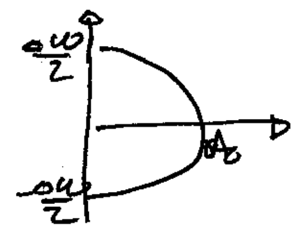
Comments

1) Many envelope/modulation functions work

Ex:

$$A(t) = A_0 \sin(\omega_2(t)t)$$

$$\omega_1(t) = \frac{\Delta\omega}{2}\cos(\omega_2(t)t)$$



where Δf is the bandwidth.

2) If a range of adiabaticity is required, optimization can help reduce pulse length.

3) Hyperbolic secant's need to be truncated, can significantly effect performance.