Lecture 2

Assignment

Read Section 5.1 Page 125-138

Homework 1 Due April 11

Today's Topics

Simple Description of Excitation

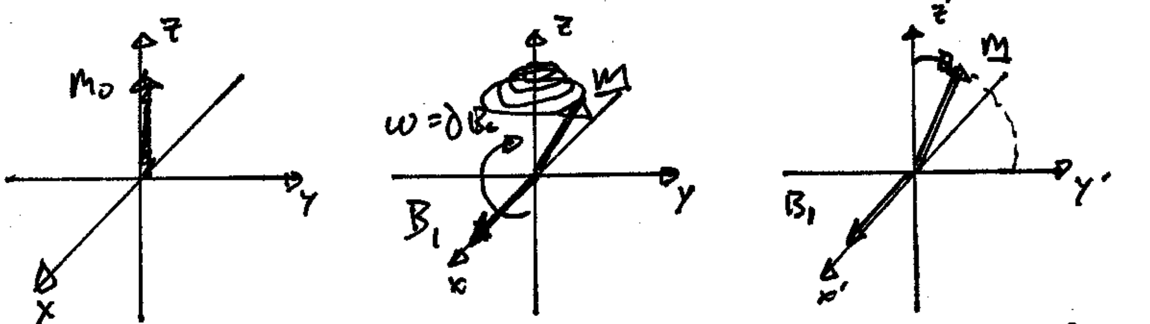
Bloch Eqn Neglecting T_1, T_2

Small-Tip-Angle Solution

Excitation k-space

Slice Selective Fourier Design

SIMPLE DESCRIPTION OF EXCITATION



POLARIZATION EQUILIBRIUM | LAB REF FRAME | ROTATING REF FRAME

MOTION OF THE MAGNETIZATION DESCRIBED BY THE BLOCH EQUATION

BLOCH EQUATION INCLUDING RELAXATION

$$\frac{d}{dt}\begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix} = \begin{pmatrix} -1/T_2 & \gamma \underline{G}\cdot\underline{r} & -\gamma B_{1,y} \\ -\gamma \underline{G}\cdot\underline{r} & -1/T_2 & \gamma B_{1,x} \\ \gamma B_{1,y} & -\gamma B_{1,x} & -1/T_1 \end{pmatrix}\begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix} + M_0 \begin{pmatrix} 0 \\ 0 \\ \frac{1}{T_1} \end{pmatrix}$$

WHERE

$\underline{G} = (G_x, G_y, G_z)$ GRADIENTS $(\underline{G}(t))$

$\underline{r} = (x, y, z)$ POSITION

FOR MOST OF THIS COURSE, WE WILL IGNORE T_1, T_2. EXCITATION IS FAST COMPARED TO RELAXATION.

BLOCH EQUATION NEGLECTING RELAXATION

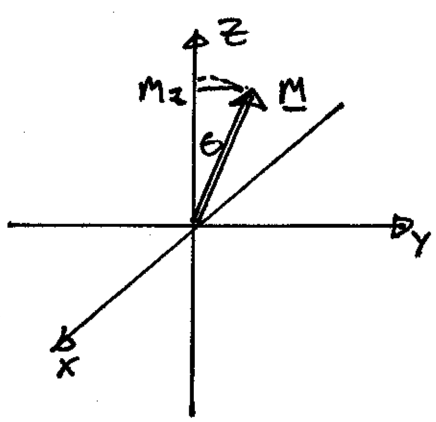
$$\frac{1}{T_1}, \frac{1}{T_2} \sim 0$$

$$\frac{d}{dt}\begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = \begin{pmatrix} 0 & \gamma \underline{G}\cdot\underline{r} & -\gamma B_{1,y} \\ -\gamma \underline{G}\cdot\underline{r} & 0 & \gamma B_{1,x} \\ \gamma B_{1,y} & -\gamma B_{1,x} & 0 \end{pmatrix}\begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix}$$

THE SOLUTION TO THIS EQUATION IS A ROTATION

WE WILL RETURN TO THIS IN 2 WEEKS!

SMALL-TIP-ANGLE APPROXIMATION



FLIP ANGLE θ SMALL MEANS

$$M_z \simeq M_0$$

FIRST TWO EQUATIONS DECOUPLE

$$\frac{d}{dt} M_x = 0 + \gamma \underline{G}\cdot\underline{r}\, M_y - \gamma B_{1,y} M_0$$

$$\frac{d}{dt} M_y = -\gamma \underline{G}\cdot\underline{r}\, M_x + 0 + \gamma B_{1,x} M_0$$

COMBINE THESE INTO A SINGLE EQN

DEFINE

$$m_{xy} = m_x + i m_y \qquad (m_{xy}(\underline{r}, t))$$

$$B_1 = B_{1,x} + i B_{1,y} \qquad (B_1(t))$$

MULTIPLY THE SECOND ($\frac{d}{dt} m_y$) EQUATION BY i, AND ADD THE TWO

$$\frac{d}{dt}(m_x + i m_y) = \gamma \underline{G} \cdot \underline{r}\, m_y - \gamma B_{1,y} m_0 - i \gamma \underline{G} \cdot \underline{r}\, m_x + i \gamma B_{1,x} m_0$$

$$= \gamma \underline{G} \cdot \underline{r} (m_y - i m_x) + \gamma m_0 (-B_{1,y} + i B_{1,x})$$

$$= \gamma \underline{G} \cdot \underline{r} (-i^2 m_y - i m_x) + \gamma m_0 (i^2 B_{1,y} + i B_{1,x})$$

$$= \gamma \underline{G} \cdot \underline{r} (-i)(m_x + i m_y) + \gamma m_0 (i)(B_{1,x} + i B_{1,y})$$

$$\frac{d}{dt} m_{xy} = -i \gamma \underline{G} \cdot \underline{r}\, m_{xy} + i m_0 \gamma B_1$$

$$\frac{d}{dt} m_{xy}(\underline{r}, t) = -i \gamma \underline{G}(t) \cdot \underline{r}\, m_{xy}(\underline{r}, t) + i m_0 \gamma B_1(t)$$

$$\frac{d}{dt} m_{xy}(\underline{r}, t) + i \gamma \underline{G}(t) \cdot \underline{r}\, m_{xy}(\underline{r}, t) = i m_0 \gamma B_1(t)$$

SIMPLE 1ST ORDER DIFFERENTIAL EQN

SOLVE USING INTEGRATING FACTOR

MULTIPLY BOTH SIDES BY

$$e^{i\int_{-\infty}^{t} \gamma \underline{G}(s)\cdot \underline{r}\, ds}$$

PRODUCING

$$\dot{m}_{xy}(\underline{r},t)\, e^{i\int_{-\infty}^{t} \gamma \underline{G}(s)\cdot \underline{r}\, ds} + i\gamma\, \underline{G}(t)\cdot \underline{r}\; m_{xy}(\underline{r},t)\, e^{i\int_{-\infty}^{t} \gamma \underline{G}(s)\cdot \underline{r}\, ds}$$
$$= i\gamma m_0 B_1(t)\, e^{i\int_{-\infty}^{t} \gamma \underline{G}(s)\cdot \underline{r}\, ds}$$

THE LEFT SIDE IS NOW AN EXACT DERIVATIVE

$$\frac{d}{dt}\left[m_{xy}(\underline{r},t)\, e^{i\int_{-\infty}^{t} \gamma \underline{G}(s)\cdot \underline{r}\, ds} \right]$$
$$= i\gamma m_0 B_1(t)\, e^{i\int_{-\infty}^{t} \gamma \underline{G}(s)\cdot \underline{r}\, ds}$$

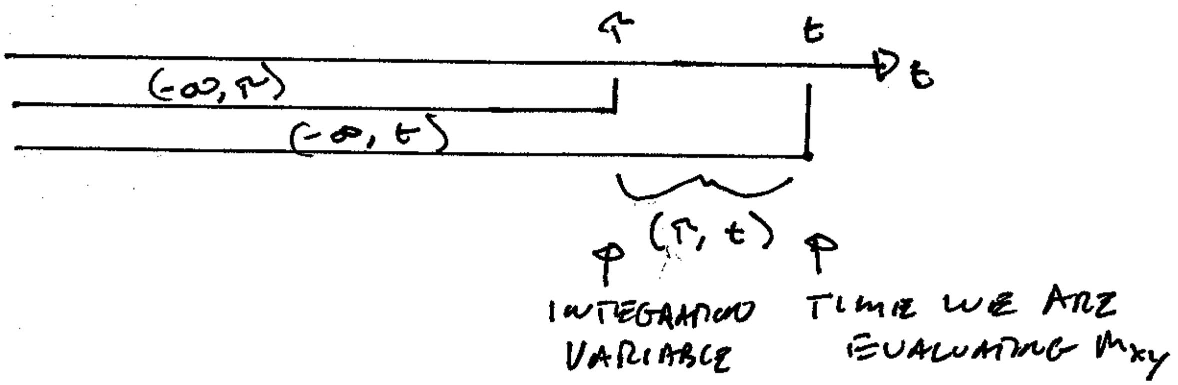
INTEGRATING BOTH SIDES

$$m_{xy}(\underline{r},t)\, e^{i\int_{-\infty}^{t} \gamma \underline{G}(s)\cdot \underline{r}\, ds} = i m_0 \int_{-\infty}^{t} \gamma B_1(\tau)\, e^{i\int_{-\infty}^{\tau} \gamma \underline{G}(s)\cdot \underline{r}\, ds}\, d\tau$$

$$m_{xy}(\underline{r},t) = i m_0\, e^{-i\int_{-\infty}^{t} \gamma \underline{G}(s)\cdot \underline{r}\, ds} \int_{-\infty}^{t} \gamma B_1(\tau)\, e^{i\int_{-\infty}^{\tau} \gamma \underline{G}(s)\cdot \underline{r}\, ds}\, d\tau$$

$$= i m_0 \int_{-\infty}^{t} \gamma B_1(\tau)\, e^{-i\int_{-\infty}^{t} \gamma \underline{G}(s)\cdot \underline{r}\, ds + i\int_{-\infty}^{\tau} \gamma \underline{G}(s)\cdot \underline{r}\, ds}\, d\tau$$

LIMITS ON THE ARGUMENTS OF EXPONENTIALS



COMBINING EXPONENTIALS

$$m_{xy}(\underline{r}, t) = i m_0 \int_{-\infty}^{t} \gamma B_1(\tau)\, e^{-i \int_\tau^t \gamma \underline{G}(s)\cdot \underline{r}\, ds}\, d\tau$$

THIS IS IN THE FORM OF A FOURIER TRANSFORM

k-SPACE FORMULATION

DEFINE

$$\underline{k}(\tau, t) = -\frac{\gamma}{2\pi} \int_\tau^t \underline{G}(s)\, ds$$

(INTEGRAL OF REMAINING GRADIENT)

DIFFERENT FROM READOUT, CONVENTION FOR EXCITATION
"EXCITATION" k-SPACE

THEN

$$m_{xy}(\underline{r}, t) = i m_0 \int_{-\infty}^{t} \gamma B_1(\tau)\, e^{i 2\pi \underline{k}(\tau, t)\cdot \underline{r}}\, d\tau$$

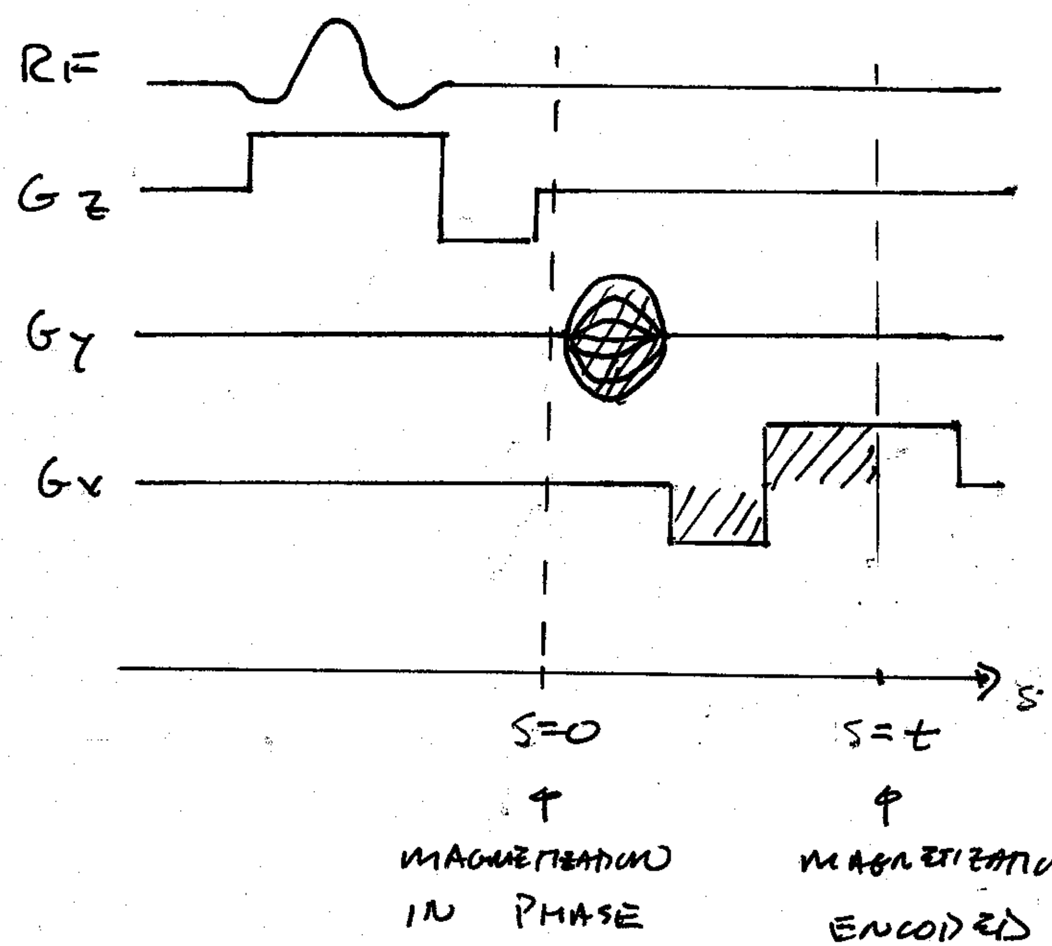
COMPARE TO READOUT K-SPACE

$$k_r(t) = \frac{\gamma}{2\pi} \int_0^t \underline{G}(s)\, ds$$

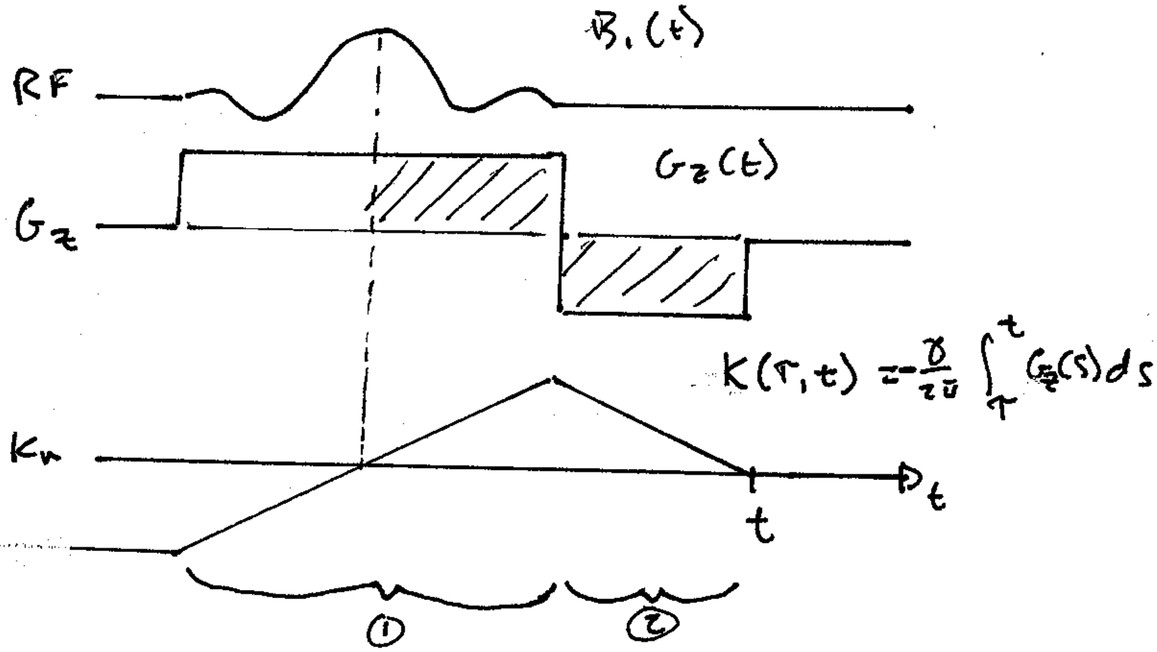
SIGNAL EQUATION

$$s(t) = \int_{\underline{R}} m_{xy}(\underline{r})\, e^{-i 2\pi \underline{k}_r(t) \cdot \underline{r}}\, d\underline{r}$$

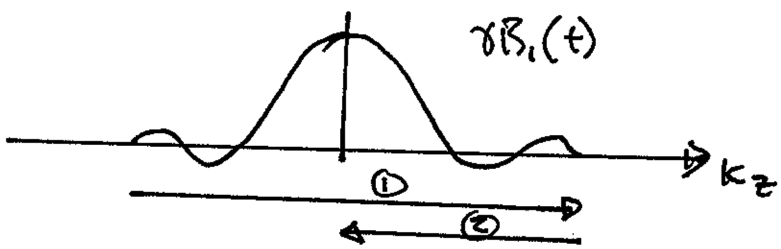
K-SPACE IS INTEGRAL OF GRADIENT FROM END OF EXCITATION TO READOUT SAMPLE



EXAMPLE: SLICE SELECTION



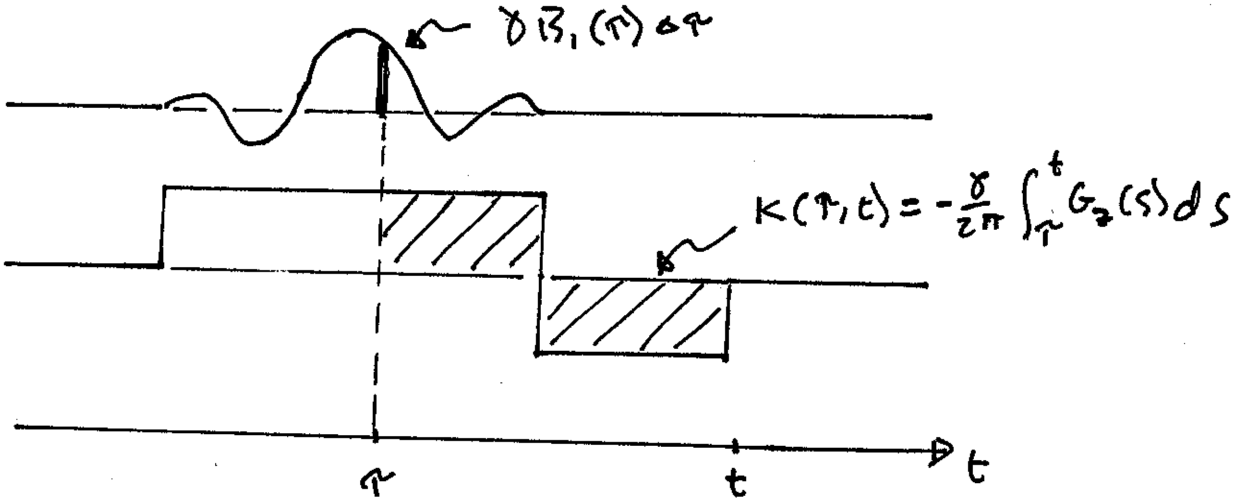
RF APPLIES A WEIGHTING IN K-SPACE DURING ①



THE REFOCUSING LOBE ② SHIFTS THE WEIGHTING BACK TO THE MIDDLE

REFOCUSES THE SLICE

GRAPHICAL DERIVATION



SMALL INCREMENT IN EXCITATION

$$\gamma B_1(\tau)\Delta\tau$$

PRODUCES SMALL INCREMENT IN MAGNETIZATION

$$\Delta m_{xy} = \left(\gamma B_1(\tau)\Delta\tau\right)(i m_0)$$

THIS PRECESSES BY

$$k(\tau,t) = -\frac{\gamma}{2\pi}\int_\tau^t G_z(s)\,ds$$

TO PRODUCE A PHASE

$$e^{i2\pi k(\tau,t) z}$$

MAGNETIZATION FROM THIS INCREMENT AT END OF PULSE

$$(i m_0)\,\gamma B_1(\tau)\Delta\tau\, e^{i2\pi k(\tau,t) z}$$

INTEGRATE OVER ALL SUCH INCREMENTS

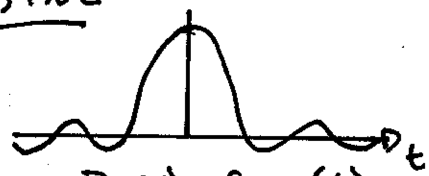
$$m_{xy}(z,t) = i M_0 \int_{-\infty}^{t} \gamma B_1(\tau)\, e^{i 2\pi k(\tau,t) z}\, d\tau$$

FOURIER DESIGN OF SLICE SELECTIVE PULSES

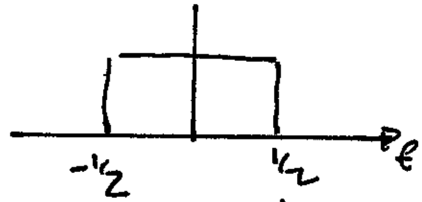
SLICE PROFILE IS FOURIER TRANSFORM OF RF PULSE, K-SPACE WEIGHTING.

CHOOSE AN RF PULSE WITH NICE TRANSFORM

SINC



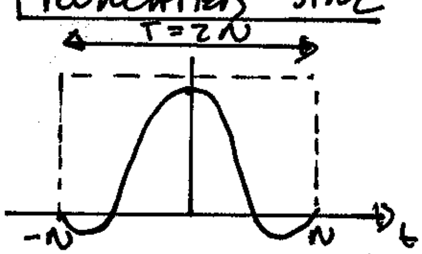
$B_1(t) = \text{sinc}(t)$



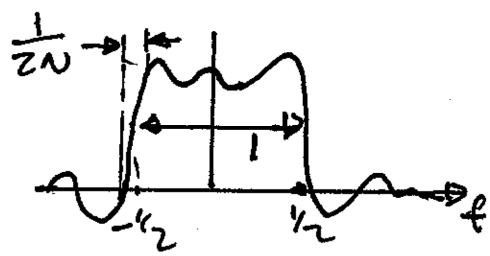
$\text{rect}(f)$

NOT PRACTICAL, SINCE $\text{sinc}(\cdot)$ CONTINUES INDEFINITELY

TRUNCATED SINC



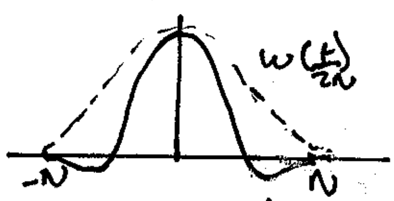
$\text{sinc}(t)\,\text{rect}(\frac{t}{2N})$



$\text{rect}(f) * 2N\,\text{sinc}(2Nf)$

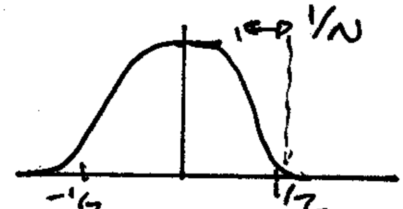
TOO MUCH RIPPLE

WINDOWED SINC



$\text{sinc}(t)\, w(\frac{t}{2N})$

$w(t)$ HAMMING WINDOW



$\text{rect}(f) * 2N\,W(2Nf)$

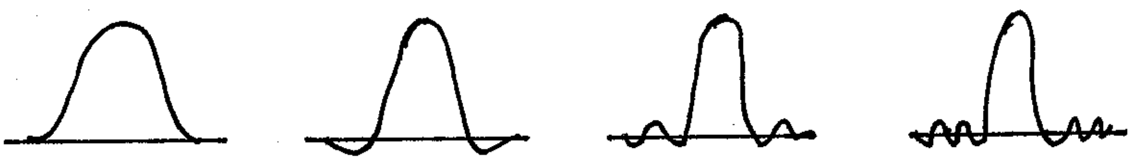
JUST RIGHT!

CHARACTERIZATION OF PULSE SHAPE

TIME - BANDWIDTH PRODUCT

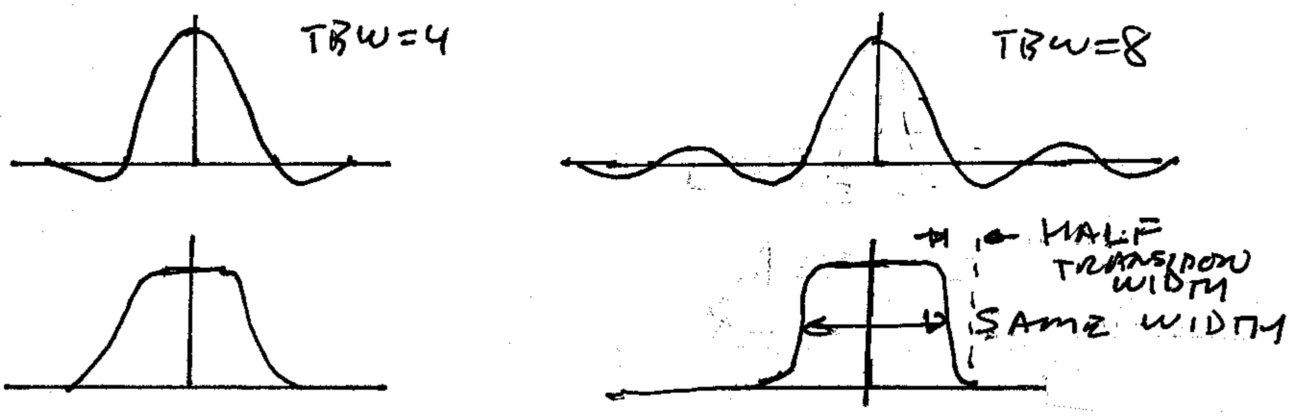
$$TBW = (2N)\,1$$
$$= 2N \qquad \text{TOTAL NUMBER OF ZEROS}$$

TYPICAL PULSES



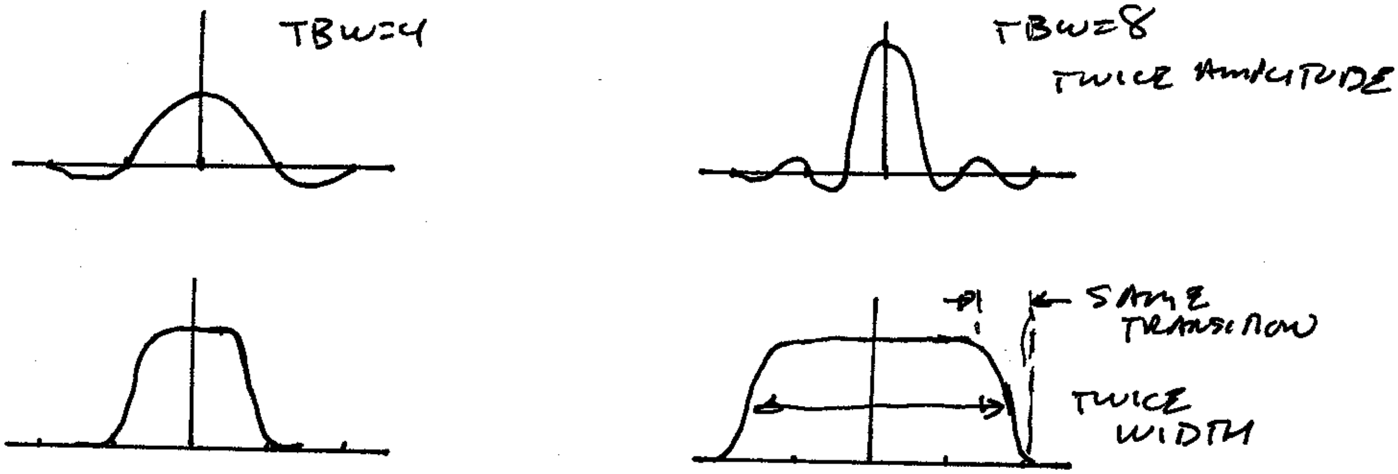
TBW=2	TBW=4	TBW=8	TBW=12
SSFP α's	180°	90°'s, α's	SAT PULSES, SLAB SELECT
msinc = 1/2	msinc = 1	msinc = 2	msinc = 3

IF WE FIX BANDWIDTH, AND MAKE T LONGER



MORE SELECTIVE PROFILE

IF WE <u>FIX DURATION</u>, AND INCREASE BANDWIDTH



WIDER EXCITATION

TYPICALLY IN MR WE <u>FIX DURATION</u>, AND <u>ADJUST THE GRADIENT AMPLITUDE</u> TO COMPENSATE FOR THE INCREASED BANDWIDTH

EXAMPLE

WE WANT A TBW=8 (msinc 2) PULSE WITH A 2ms DURATION.

IF THE SLICE THICKNESS IS 1cm, WHAT IS THE GRADIENT AMPLITUDE?

ANSWER:

$$(T)(BW) = 8$$
$$(2ms)(BW) = 8$$
$$BW = 4\ kHz$$

WE WANT THIS TO CORRESPOND TO A SLICE THICKNESS $\Delta z = 1\ cm$

$$\frac{\gamma}{2\pi} G \Delta z = 4\ kHz$$

$$(4.257\ kHz/G)\, G\, (1cm) = 4\ kHz$$

$$\underline{\underline{G = 0.94\ G/cm}}$$