Lecture 5

ASSIGNMENT

READ "EPI PULSES" PAGES 129-133
"EDDY CURRENT COMPENSATION" PAGES 316-331

LAST TIME

2D SPIRAL PULSES

TODAY

PRACTICAL ISSUES IN 2D SPIRAL PULSE DESIGN

NEXT

EPI AND SPECTRAL SPATIAL PULSES

SUMMARY OF 2D SPIRAL PULSE DESIGN

1) DESIGN A SPIRAL K-SPACE TRAJECTORY $\underline{k}(t)$

RESOLUTION $\quad \Delta r = \frac{1}{2k_{max}}$

FOV $\quad FOV = 2N\Delta r \quad$ (N TURN SPIRAL)

2) DESIGN GRADIENT WAVEFORM

$$\underline{G}(t) = \frac{d}{dt}\underline{k}(\tau(t))$$

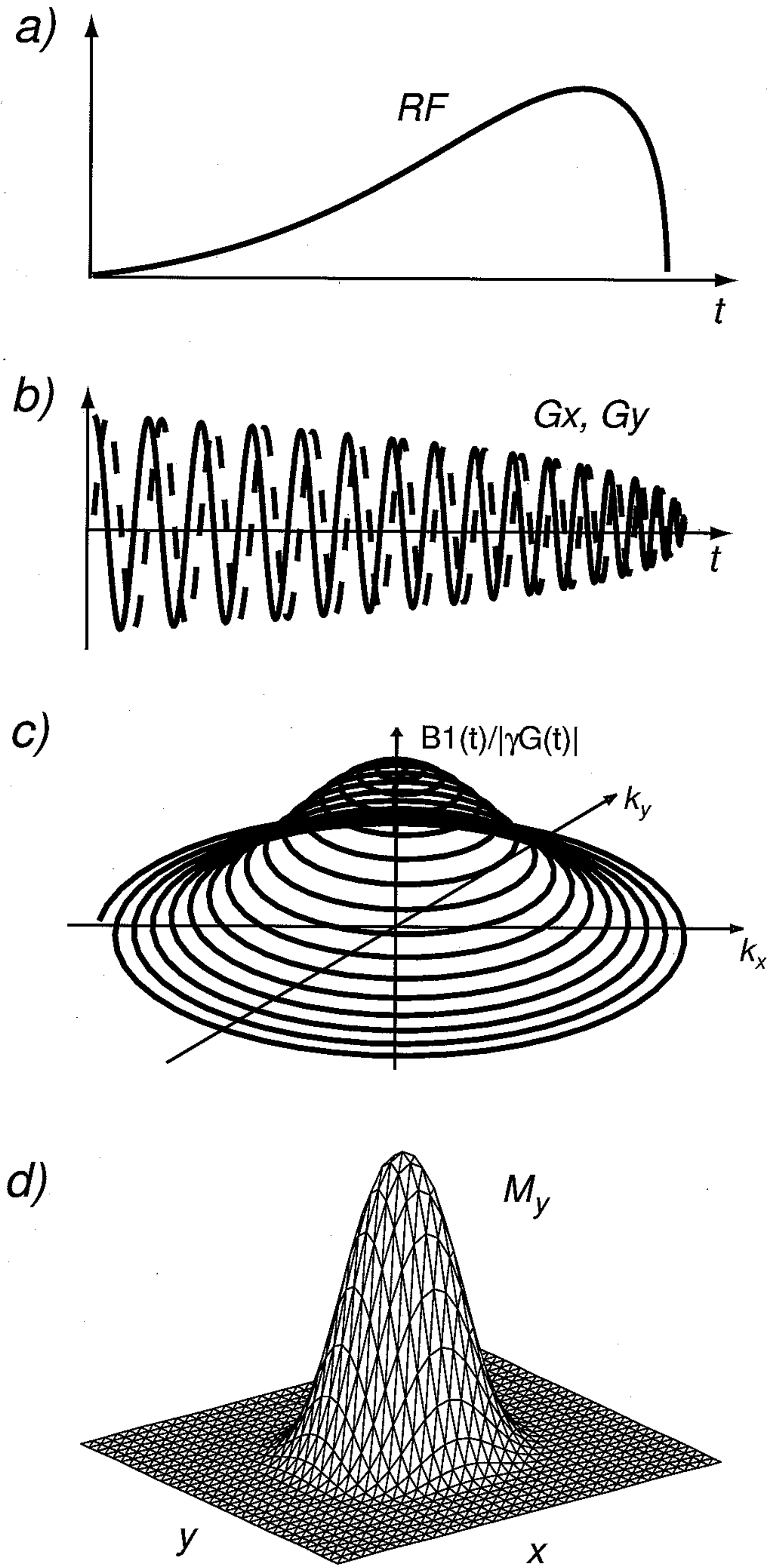
OPTIMIZE $\tau(t)$ TO MEET AMPLITUDE AND SLEW RATE LIMITS

3) DESIGN K-SPACE WEIGHTING $W(\underline{k})$ AS THE FOURIER TRANSFORM OF THE VOLUME TO BE EXCITED

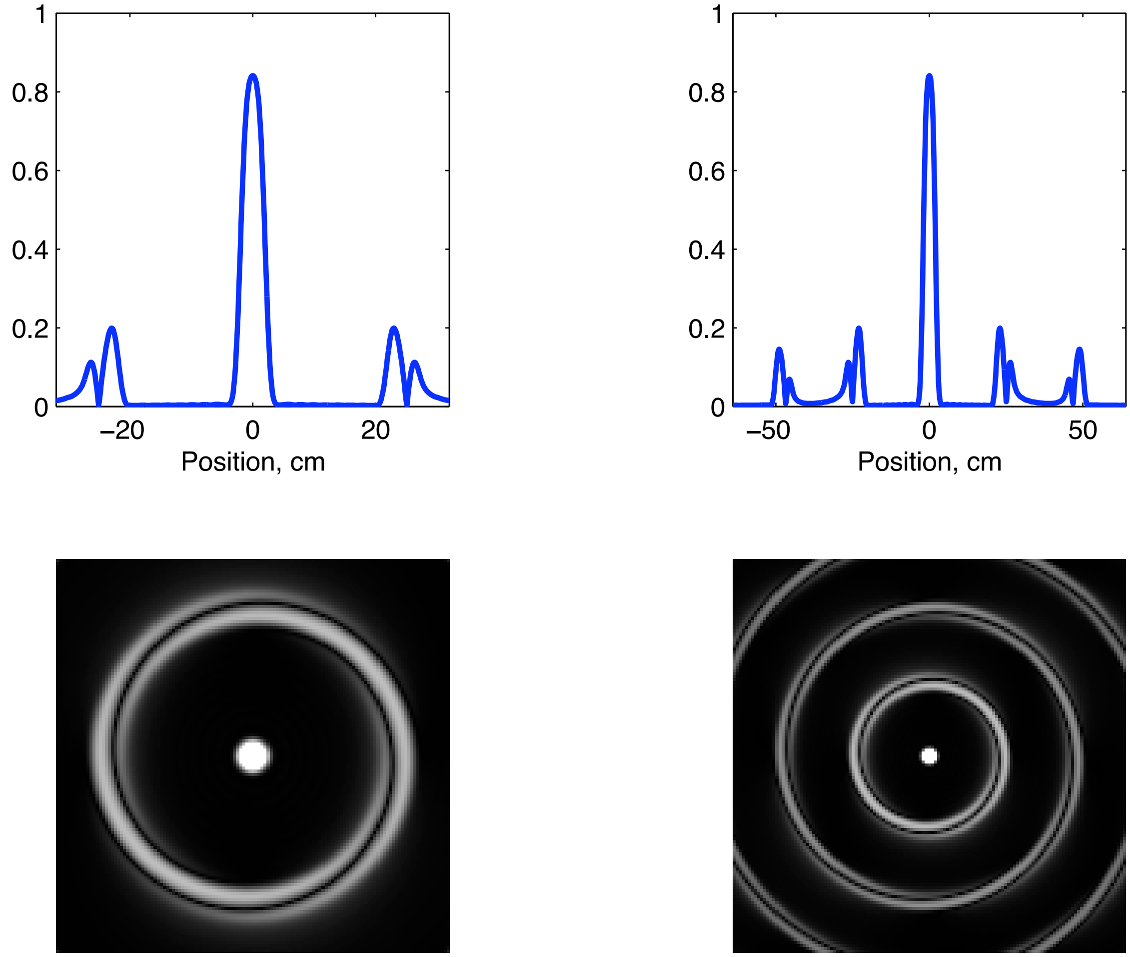
4) CALCULATE THE RF WAVEFORM

$$B_1(t) = |G(t)|\, W(\underline{k}(\tau(t)))$$

THEN SCALE TO THE DESIRED FLIP ANGLE


a)
RF
t
b)
Gx, Gy
t
c)
B1(t)/|γG(t)|
k_y
k_x
d)
M_y
y
x

Sidelobe Geometry



12 turn spiral, 4.5 ms, constant slew rate
1 cm resolution (kmax = +/- 0.5 cycles/cm)
SBW = 4
WIndowed jinc RF

Sidelobes at +/- 24 cm, +/- 48 cm

BETTER DENSITY COMPENSATION

EXCITED VOLUME

$$m_{xy}(\underline{r},t) = i m_0 \int_{\underline{k}} P(\underline{k})\, e^{i 2\pi \underline{k}\cdot\underline{r}}\, d\underline{k}$$

WHERE

$$P(\underline{k}) = \int_{-\infty}^{t} \underbrace{\frac{\gamma B_1(\tau)}{|k'(\tau,t)|}}_{W(\underline{k})}\; \underbrace{{}^3\delta(\underline{k}(\tau,t)-\underline{k})\,|k'(\tau,t)|\, d\tau}_{\substack{\text{UNIT STRENGTH DELTA} \\ S(\underline{k})}}$$

THE VALUE OF $|k'(\tau,t)|$ IS A APPROXIMATION OF THE DENSITY COMPENSATION FUNCTION FOR $\underline{k}(\tau,t)$

WHEN $|k'(\tau,t)|$ IS SMALL, WE NEED $B_1(\tau)$ TO ALSO BE SMALL TO PRODUCE CORRECT $W(\underline{k})$

IF WE HAVE AN ESTIMATE OF THE DENSITY $d(t)$ FROM SOME ALGORITHM (VORONOI, FOR EXAMPLE), THE DENSITY COMPENSATION FUNCTION IS $\frac{1}{d(t)}$ AND

$$|k'(\tau,t)| \sim \frac{1}{d(\tau)}$$

THEN

$$W(\underline{k}(\tau,t)) = \frac{\gamma B_1(\tau)}{1/d(\tau)}$$

$$B_1(\tau) = \frac{1}{\gamma}\,\frac{W(\underline{k}(\tau,t))}{d(\tau)}$$

WHERE $d(\tau)$ IS LOCAL SAMPLE DENSITY.

SHIFTING THE VOLUME

EXCITED VOLUME

$$m_{xy}(\underline{r},t) = i m_0 \int_{-\infty}^{t} \gamma B_1(\tau) e^{i 2\pi \underline{k}(\tau,t)\cdot \underline{r}} d\tau$$

WITH MODULATED RF

$$B_{1,m}(\tau) = B_1(\tau) e^{-i 2\pi \underline{k}(\tau,t)\cdot \underline{r_0}}$$

WE GET

$$m_{xy,m}(\underline{r},t) = i m_0 \int_{-\infty}^{t} \gamma B_1(\tau) e^{-i 2\pi \underline{k}(\tau,t)\cdot \underline{r_0}} e^{+i 2\pi \underline{k}(\tau,t)\cdot \underline{r}} d\tau$$

$$= i m_0 \int_{-\infty}^{t} \gamma B_1(\tau) e^{i 2\pi \underline{k}(\tau,t)\cdot(\underline{r}-\underline{r_0})} d\tau$$

$$= m_{xy}(\underline{r}-\underline{r_0}, t)$$

SAME PROFILE, SHIFTED TO $\underline{r}_0$.

NOTE THAT WE CAN APPLY

$$\phi(\tau) = -2\pi \underline{k}(\tau,t)\cdot \underline{r_0}$$

TO THE <u>PHASE</u> CHANNEL, OR

$$\omega(\tau) = \phi'(\tau) = \frac{d}{d\tau} -2\pi \underline{k}(\tau,t)\cdot \underline{r_0}$$

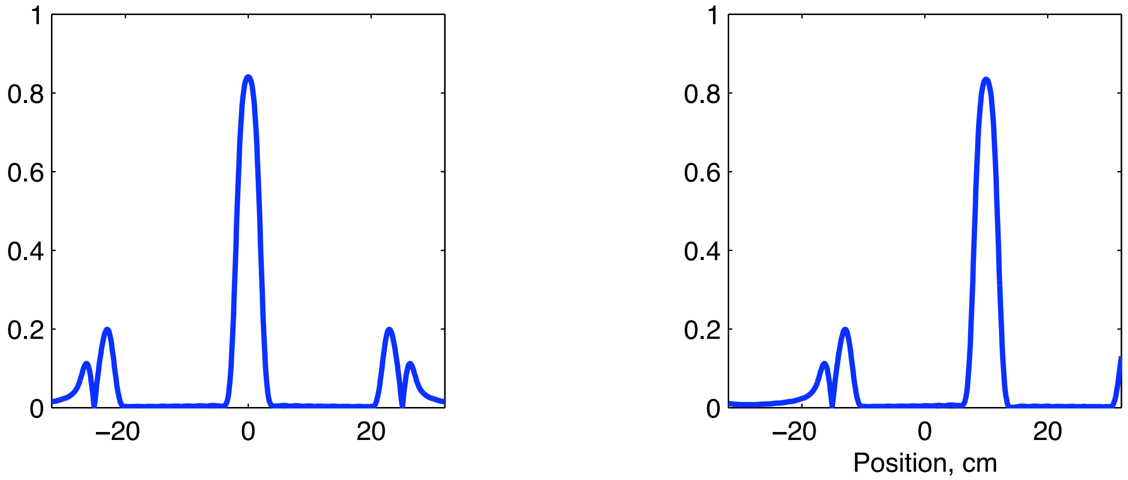
$$= \frac{d}{d\tau}\left(-2\pi \frac{\gamma}{2\pi}\int_{\tau}^{t} G(s)\,ds\right)\cdot \underline{r_0}$$

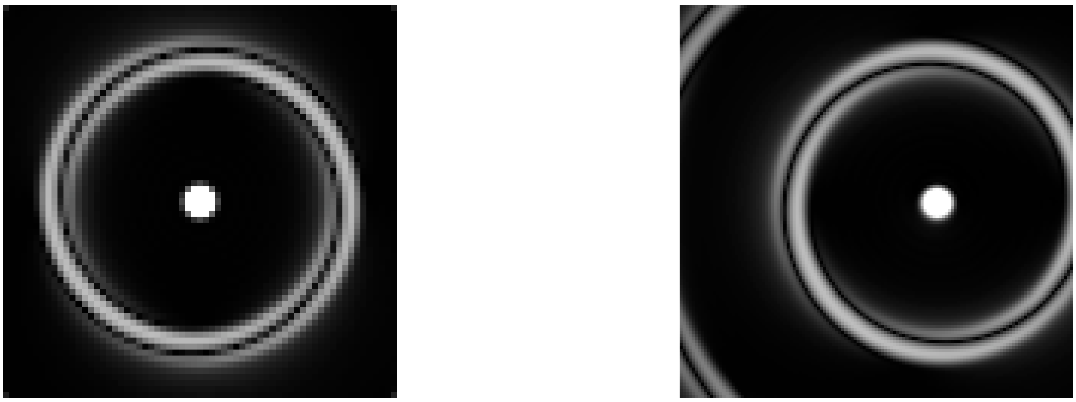
$$= \gamma \underline{G}(\tau)\cdot \underline{r_0}$$

TO THE <u>FREQUENCY</u> CHANNEL.

Modulation to Shift FOV

$$B_{1,m}(\tau) = B_1(\tau)e^{-i2\pi \underline{k}(\tau,t)\cdot \underline{r}_0}$$





12 turn spiral
1 cm resolution (kmax = +/- 0.5 cycles/cm)
SBW = 4
WIndowed jinc RF
Modulated to +10 cm along x

PRACTICAL CONCERNS: GRADIENT DELAYS

TIME LAG BETWEEN WHEN GRADIENT WAVEFORM
IS REQUESTED, AND WHEN SOMETHING HAPPENS
IN THE MAGNET BORE

MANY CAUSES, ALL LUMPED TOGETHER

- DIGITAL DELAYS IN WAVEFORM GENERATION
- FREQUENCY RESPONSE OF GRADIENT COIL / AMPLIFIER
- EDDY CURRENTS

TYPICAL NUMBERS ARE 40 us TO 150 us ON
A WHOLE BODY SYSTEM.

WHAT DOES THIS DO TO 2D SPIRAL
EXCITATION PULSES?

CONSTANT ANGULAR RATE SPIRAL

ASSUME WE DESIGNED A 2D SPIRAL PULSE WITH THE CONSTANT ANGULAR RATE TRAJECTORY

$$\underline{k}(t) = k_{max}\left(1 - \frac{t}{T}\right) e^{i 2\pi N(1 - t/T)}$$

AND AN ASSOCIATED $B_1(t)$.

IF THE GRADIENT IS DELAYED BY A TIME δ

$$\underline{k}(t-\delta) = k_{max}\left(1 - \frac{t-\delta}{T}\right) e^{i 2\pi N\left(1 - \frac{t-\delta}{T}\right)}$$

IF δ IS SMALL

$$\underline{k}(t-\delta) \simeq k_{max}\left(1 - \frac{t}{T}\right) e^{i 2\pi N\left(1 - \frac{t}{T}\right)} e^{i 2\pi N \delta / T}$$

$$= \underline{k}(t)\, e^{i 2\pi N \delta / T}$$

RECALL THAT

$$\underline{k}(t) = k_x(t) + i k_y(t)$$

MULTIPLICATION BY $e^{i 2\pi N \delta / T}$ IS A <u>ROTATION</u> OF THE K-SPACE TRAJECTORY BY

$$\underline{\Theta = 2\pi N \delta / T} \quad \text{RADIANS}$$

EXAMPLE: 12 TURN SPIRAL, 4.5 ms LONG
GRADIENT DELAY OF 16 μs

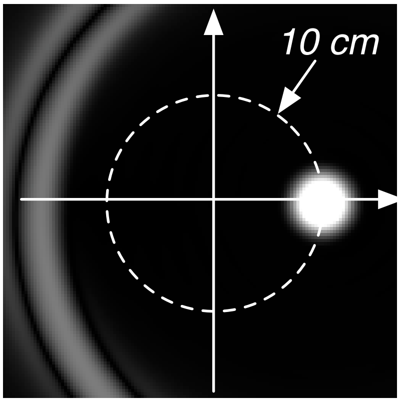
$$\theta = 2\pi(12)\frac{0.016}{4.5} = 0.27 \text{ RADIANS}$$

$$= \underline{\underline{15^\circ}}$$

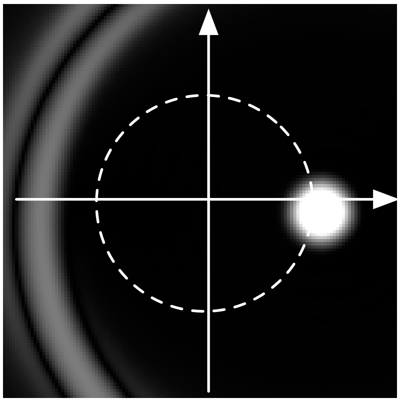
RESULT: GRADIENT DELAY CAUSES ROTATION ABOUT THE ISOCENTER PRIMARILY, WITH LITTLE OTHER DISTORTION

Effect of Gradient Delay: Constant Angular Rate Spiral

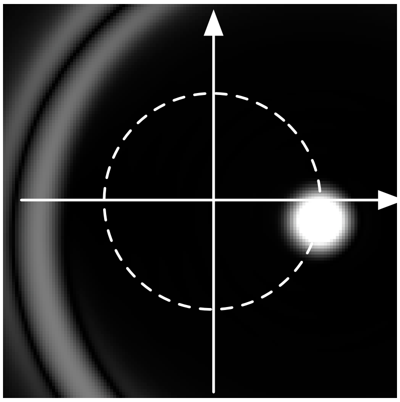
No Delay



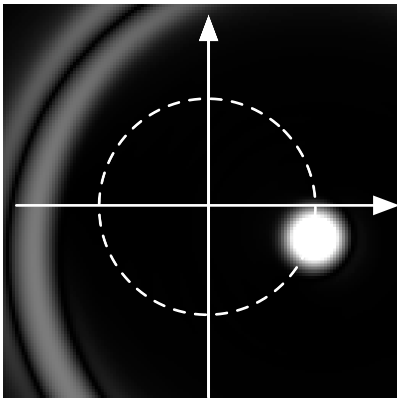
4 us



8 us



16 us



12 turn spiral, 6.5 ms, constant angular rate
1 cm resolution (kmax = +/- 0.5 cycles/cm)
SBW = 4
WIndowed jinc RF
Modulated to +10 cm along x

Delay Produces:
Rotation about isocenter

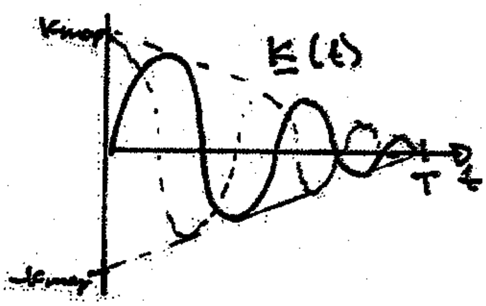
GRADIENT / SLEW OPTIMIZED SPIRAL

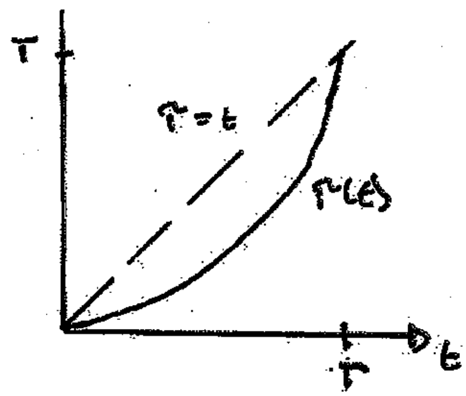
NOW

$$\underline{k}(\tau(t)) = k_{max}\left(1 - \frac{\tau(t)}{T}\right) e^{i2\pi N\left(1 - \frac{\tau(t)}{T}\right)}$$

LET δ BE THE GRADIENT DELAY

$$\underline{k}(\tau(t-\delta)) = k_{max}\left(1 - \frac{\tau(t-\delta)}{T}\right) e^{i2\pi N\left(1 - \frac{\tau(t-\delta)}{T}\right)}$$





TYPICAL $\tau(t)$

IF δ IS SMALL, $\tau(t)$ SMOOTH, SLOWLY VARYING

$$\underline{k}(\tau(t-\delta)) \simeq k_{max}\left(1 - \frac{\tau(t)}{T}\right) e^{i2\pi N\left(1 - \frac{\tau(t) - \delta\tau'(t)}{T}\right)}$$

$$= k_{max}\left(1 - \frac{\tau(t)}{T}\right) e^{i2\pi N\left(1 - \frac{\tau(t)}{T}\right)} e^{i2\pi N\delta\tau'(t)/T}$$

$$= \underline{k}(\tau(t))\, e^{i2\pi N\delta\tau'(t)/T}$$

AGAIN, THIS IS A ROTATION BUT IS DEPENDENT ON TIME / SPATIAL FREQUENCY

EACH SPATIAL FREQUENCY ROTATED BY A DIFFERENT AMOUNT $\Rightarrow$ DISPERSION

ROTATION ANGLE

$$\Theta(t) = 2\pi N \delta \tau(t)/T \text{ RADIANS}$$

RESULT:

HIGH SPATIAL FREQUENCIES

LARGE GRADIENT AMPLITUDES

PLAYED OUT SLOWER

LESS ROTATION

LOW SPATIAL FREQUENCIES

SMALL GRADIENT AMPLITUDES

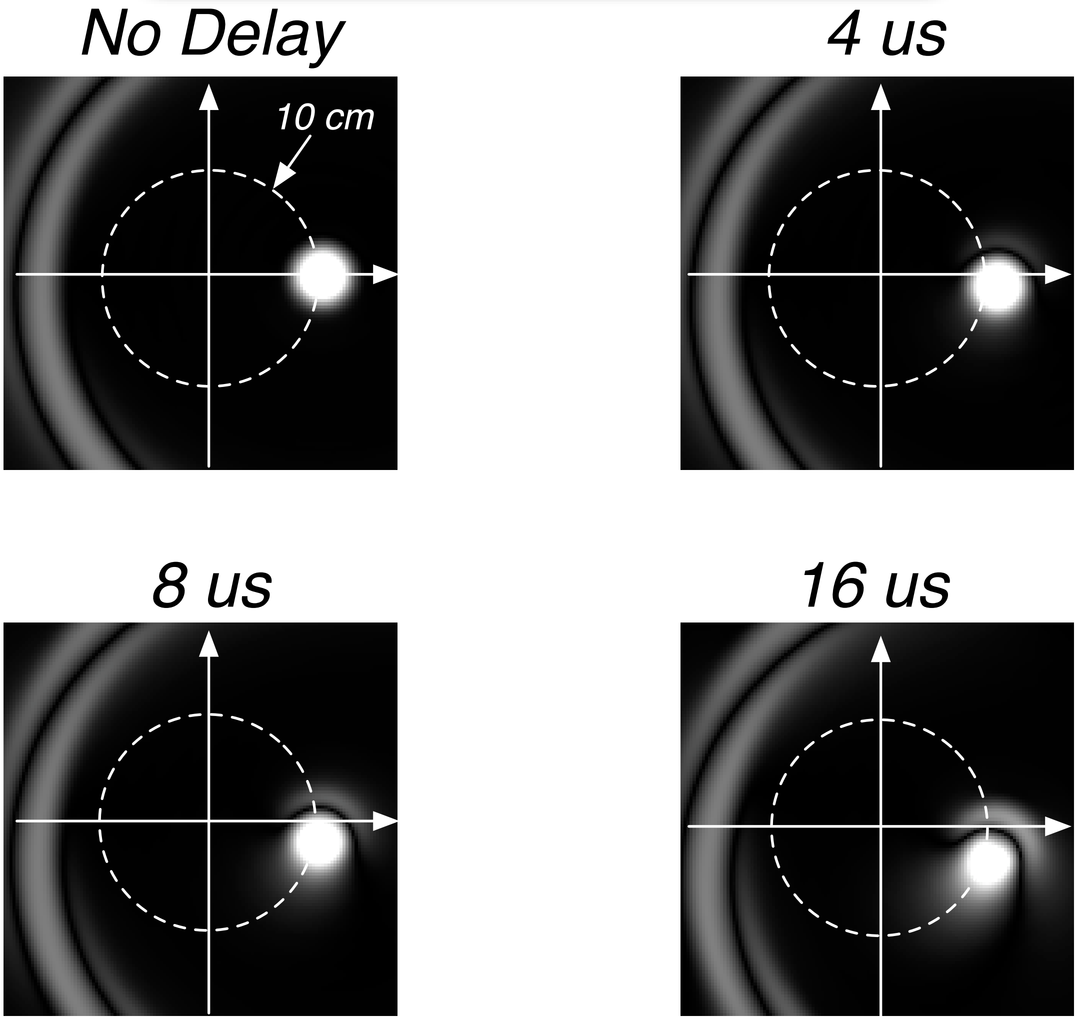
PLAYED OUT FASTER

MORE ROTATION

HIGH AND LOW SPATIAL FREQUENCIES

GET OUT OF ALIGNMENT

Effect of Gradient Delay: Constant Slew Rate Spiral



12 turn spiral, 4.5 ms, constant slew rate
1 cm resolution (kmax = +/- 0.5 cycles/cm)
SBW = 4
WIndowed jinc RF
Modulated to +10 cm along x

Delay Produces:
Rotation about isocenter
Distortion of the selected volume

OFF-RESONANCE EFFECTS

MANY SOURCES OF B_0 VARIATIONS

- MAIN FIELD INHOMOGENEITY
- SUBJECT SUSCEPTIBILITY
- CHEMICAL SHIFT
- EDDY CURRENTS

HOW DO THESE EFFECT 2D SPIRAL PULSES?

IDEALLY THE K-SPACE WEIGHTING IS

$$w(\underline{k}(t)) = \frac{\gamma B_1(t)}{|\gamma \underline{G}(t)|}$$

IF THE RF IS APPLIED AT AN OFF-RESONANCE FREQUENCY ω, THE ACTUAL WEIGHTING IS

$$w_a(\underline{k}(t)) = \frac{\gamma B_1(t)\, e^{-i\omega t}}{|\gamma \underline{G}(t)|}$$

FOR A CONSTANT ANGULAR RATE INWARD SPIRAL

$$\underline{k}(t) = k_{max}\left(1-\frac{t}{T}\right) e^{i 2\pi N(1-t/T)}$$

$$|\underline{k}(t)| = k_{max}\left(1-\frac{t}{T}\right)$$

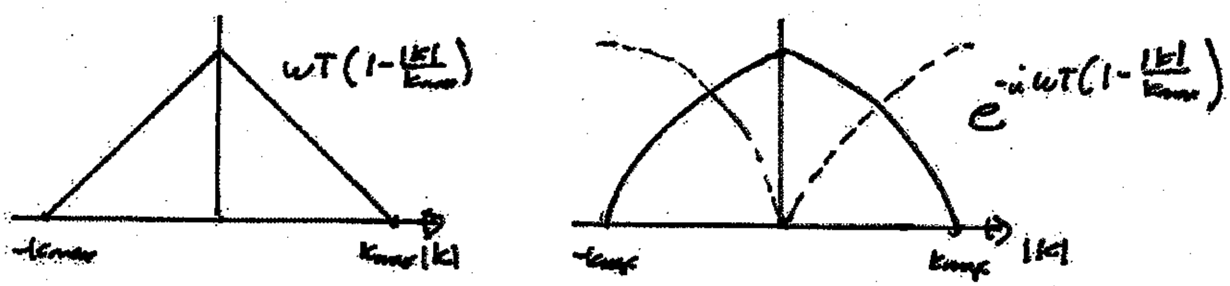
$$t = T\left(1-\frac{|k|}{k_{max}}\right)$$

THEN

$$w_a(\underline{k}(t)) = \frac{\gamma B_1(t)}{|\gamma G(t)|} e^{-i\omega T\left(1-\frac{|k|}{k_{max}}\right)}$$

$$w_a(\underline{k}) = \underbrace{w(\underline{k})}_{\text{DESIRED WEIGHTING}} \underbrace{e^{-i\omega T\left(1-\frac{|k|}{k_{max}}\right)}}_{\text{PHASE ABERRATION}}$$

EFFECT OF PHASE ABERRATION



SMALL AMOUNT OF ABERRATION (LESS THAN SBW/4 CYCLES)

BLURRING OF IMPULSE RESPONSE

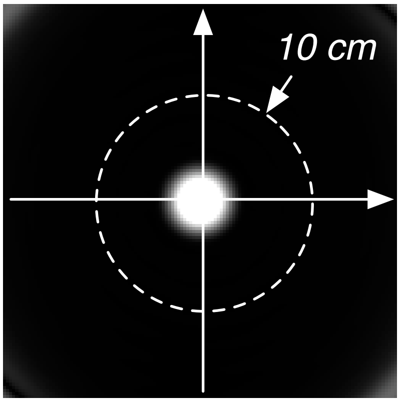
SIGNIFICANT ABERRATION (MORE THAN SBW/4 CYCLES)

SIGNIFICANT BLURRING

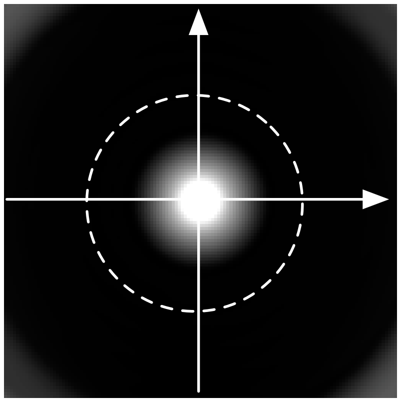
LOSS OF AMPLITUDE

Effect of Off-Resonance

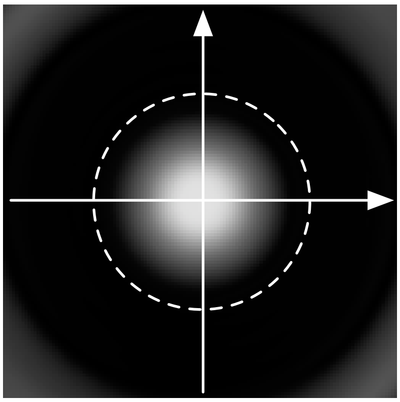
On Resonance



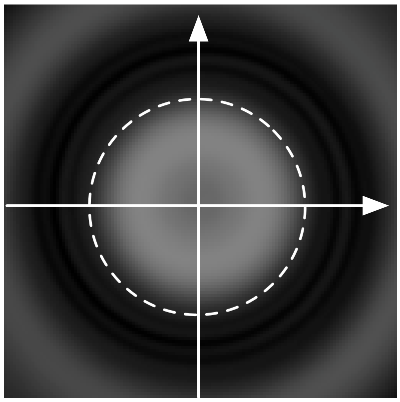
220 Hz



440 Hz

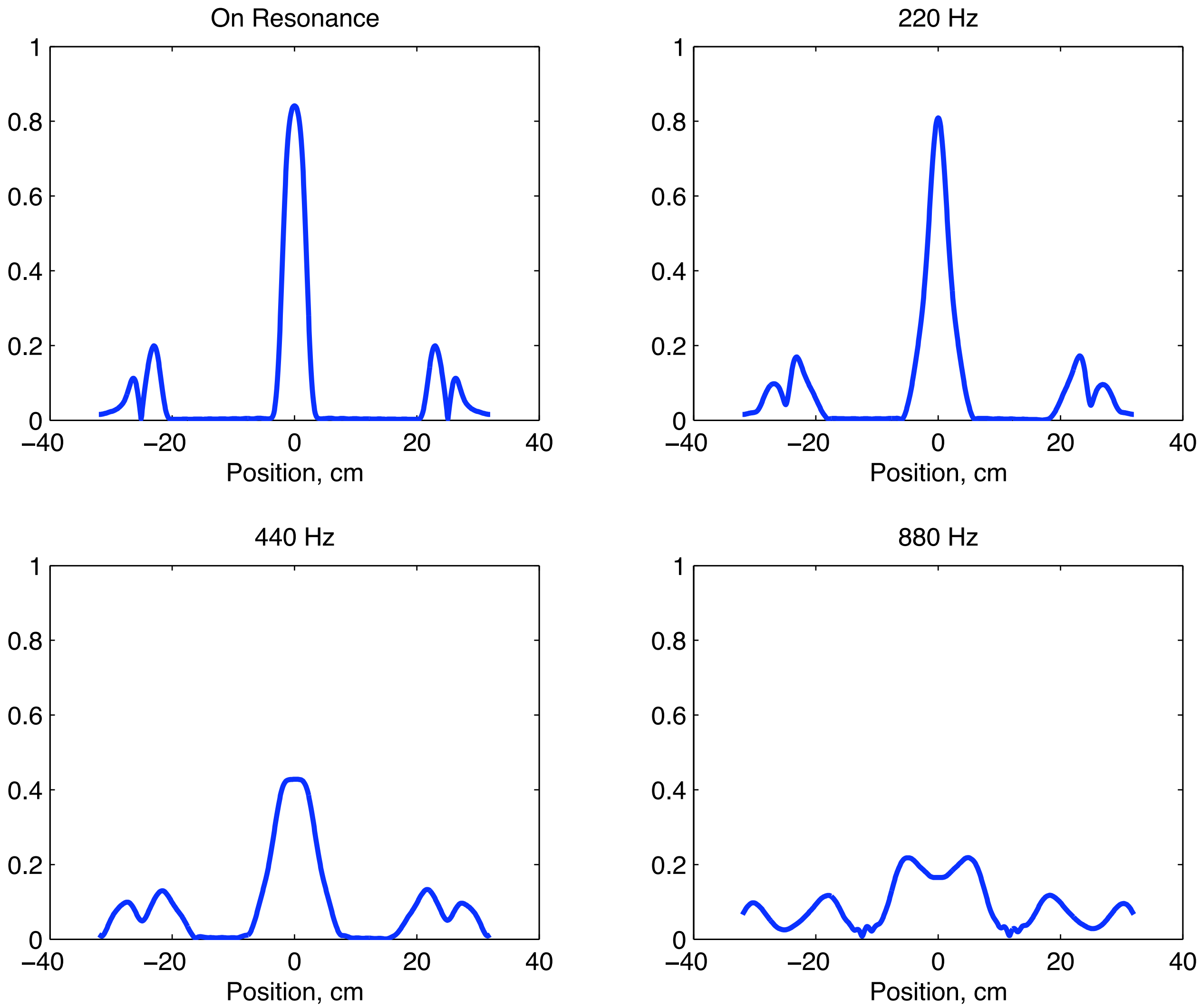


880 Hz



12 turn spiral, 4.5 ms, constant slew rate
1 cm resolution (kmax = +/- 0.5 cycles/cm)
SBW = 4
WIndowed jinc RF
220 Hz is one cycle over pulse length

Effect of Off-Resonance



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1 cm resolution (kmax = +/- 0.5 cycles/cm)
SBW = 4
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220 Hz is one cycle over pulse length